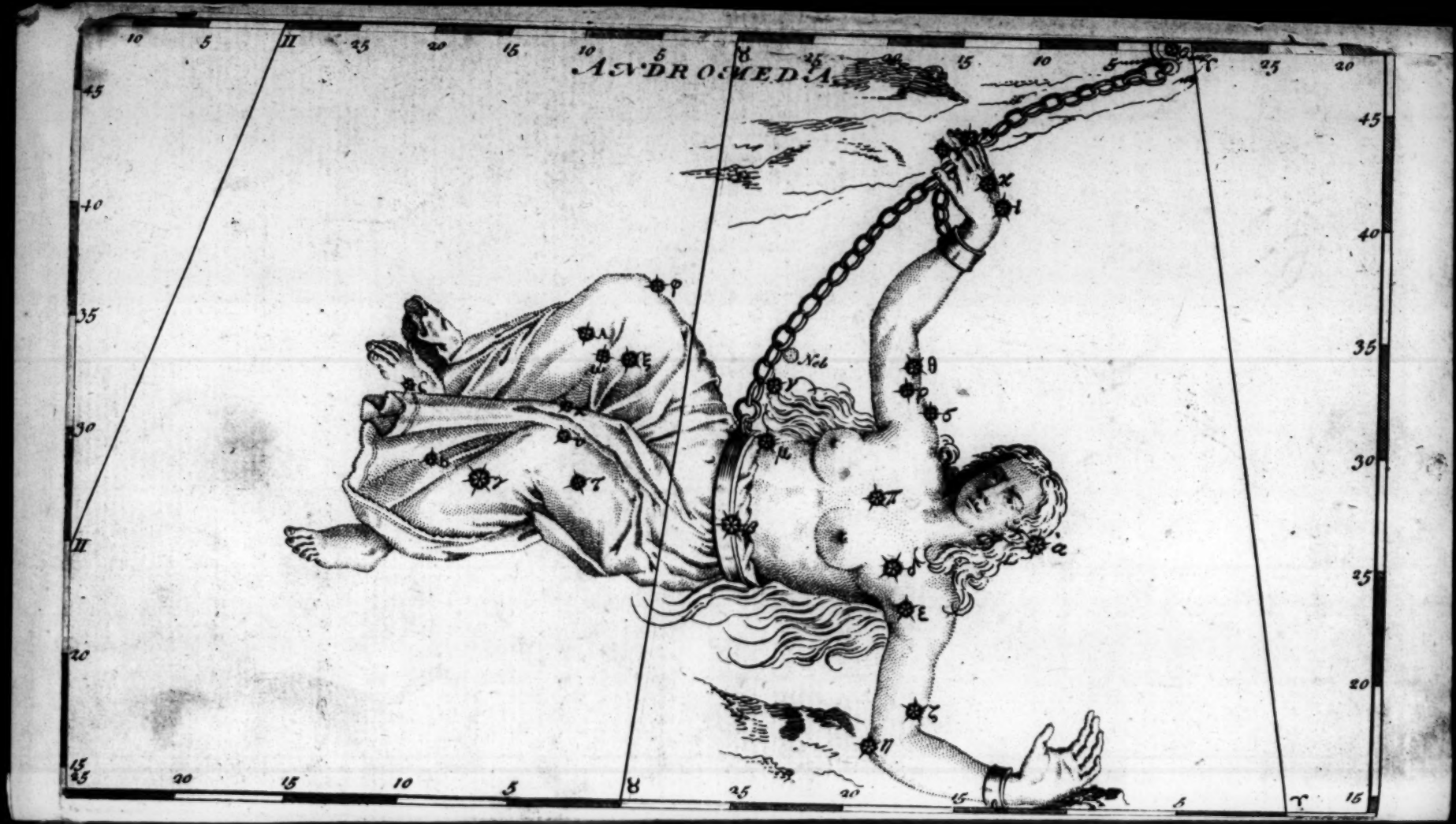
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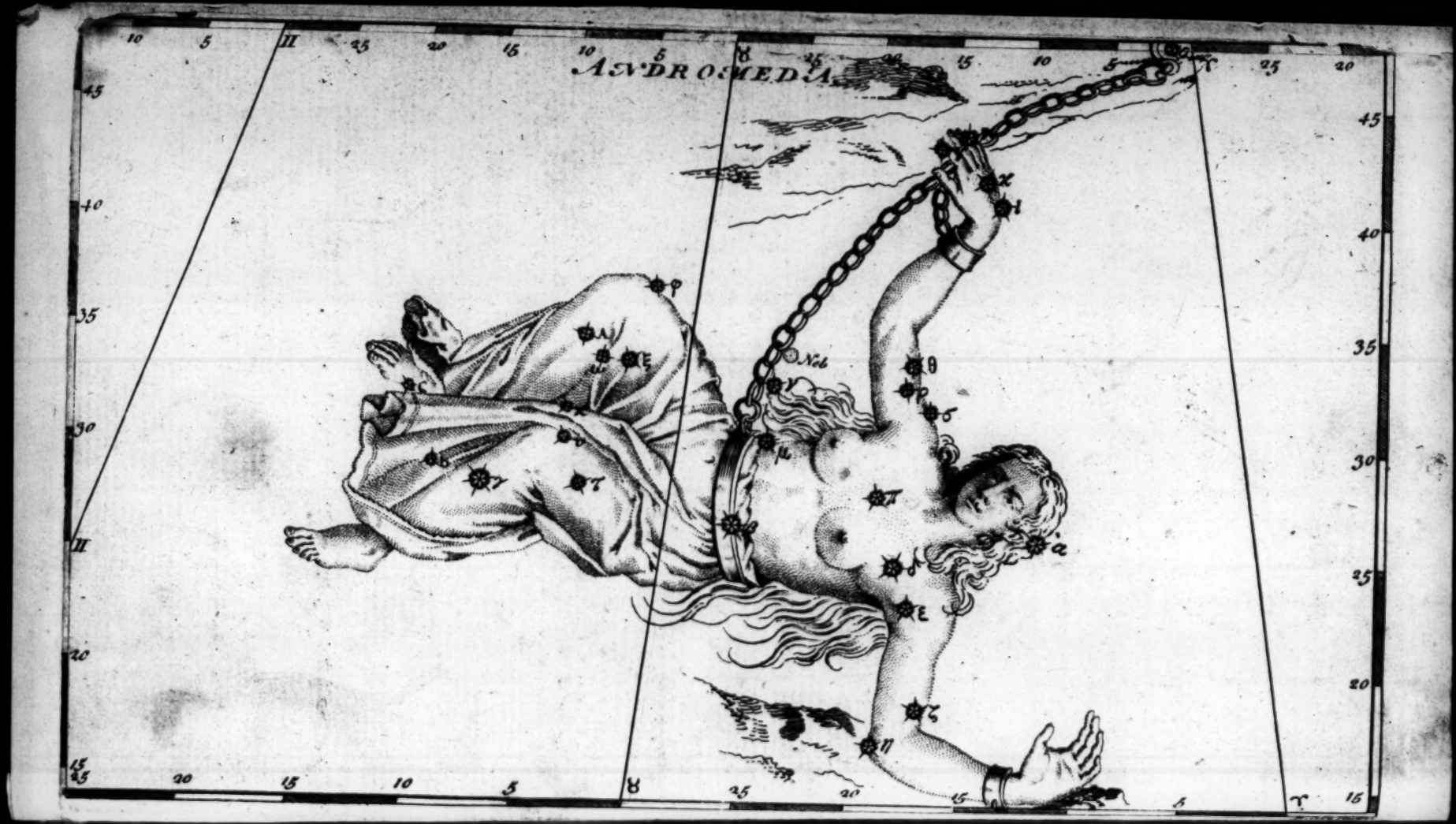
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MATHEMATICAL EXERCISES:

CONTAINING,

I. The Principles of the Orthographick Projection of the Sphere; with the Application thereof to the Solution of ſome Problems in Aſtronomy, and the Demonſtration of certain Theorems of great Uſe in Spherical Trigonometry.

II. The Principles of the Stereographick Projection, and the Application thereof to the Deſcribing of the Repreſentations of the Circles, *&c.* of the Sphere on the Planes of different Great Circles.

III. Sixteen new Problems, to be anſwered in the Second Number.

By *JOHN TURNER.*

N°. I.

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(Price ONE SHILLING.)

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ADVERTISEMENT.

ALL Perſons, inclining to encourage this Work, are deſired to ſend their Performances (whether new Problems, Paradoxes, Solutions, *&c.*) Poſt paid, to be left with Mr. *James Morgan*, at the *Three-Cranes*, in *Thames-ſtreet*, before the 20th of *February*, 1751:

The ſecond Number being to be publiſhed on the 25th of *March* enſueing.

THE

PREFACE.

IT may, perhaps, be thought unneceſſary to trouble the World with this Performance, at a Time, when ſo many of the like Kind are already extant. But, if we conſider the Imperfections which moſt of them lie under, either thro' Want of Ability or Care in the Authors, we hope what is here offered to the Public will not prove unacceptable; as we ſhall exert our beſt Endeavours to put every Thing in as clear Light as poſſible, and to oblige the lower, as well as the higher Claſs of Readers.

In the Courſe of this Performance, we intend, beſides the Problems and their Solutions, to give in each Number ſome original Paper or uſeful Eſſay on ſuch Branches of the Mathematics, which, we apprehend, have not been ſufficiently explained. Conformable to which Deſign, we have, in this our firſt Number, given the Principles of the Orthographick Projection, and the Application thereof to the Solution of ſome Problems in Aſtronomy, together with the Demonſtration of certain Theorems of great Uſe in Spherical Trigonometry; which we

we believe is done in a Manner entirely new.

The Authors, who have wrote on the Projection of the Sphere, have attended chiefly to the Stereographick Projection, and touched but ſlightly on the Orthographick on Account of the Difficulty in deſcribing Ellipſes, which are the Repreſentations of Oblique Circles. But, in the following Pages, we hope it will appear that there is no Neceſſity for the Deſcription of Ellipſes, in the Uſe of the Orthographick Projection, in Problems of the Sphere, where the Work may be performed by Right-lines only; whereas both Right-lines and Circles are requiſite in the Stereographick Projection. Not but that we imagine the Principles of the Stereographick Projection, as Here laid down, will appear altogether as eaſy and perſpicuous, as in thoſe Authors who have wrote much more copiouſly on the Subject.

We have no more to add than that we ſhall, upon all Occaſions, ſhew our Willingneſs to oblige our Correſpondents by inſerting their Performances, whether Problems or Solutions, but, at the ſame Time, take Care that Nothing ſhall appear to their Diſadvantage.

THE

THE PROJECTION OF THE SPHERE.

DEFINITIONS.

1. A SPHERE is a Solid, whoſe Surface is every where equidiſtant from a Point within the Sphere, called its Center.

It appears, from this Definition, *that, if a Sphere be cut by a Plane, the Section will be a Circle.*

2. Thoſe Circles, whoſe Planes paſs thro' the Center of the Sphere, are called Great-Circles.

3. All other Circles, whoſe Planes do not paſs thro' the Center of the Sphere, are called Leſſer-Circles.

4. Any Right-line, paſſing thro' the Center and terminating in the Surface of the Sphere, is called

an Axis; and the two Extremities of an Axis are called the Poles of all the Circles whoſe Planes are perpendicular to the ſaid Axis.

5. The Repreſentation, or Appearance, of any Object upon a Plane, is the Figure formed by Rays paſſing from the Eye or Point of Sight thro' every Point of the ſaid Object.

6. The Projection of the Sphere is the Repreſentation of the Circles thereof upon a Plane, called the Plane of Projection; and is divided into the Orthographick, the Stereographick, the Gnomonick, *&c.*

7. The Orthographick Projection is the Repreſentation of the Circles of the Sphere, upon the Plane of a Great Circle, by Lines drawn, perpendicular to the ſaid Plane, from every Point in the Circles to be projected.

8. The Stereographick Projection is the Repreſentation of the Circles of the Sphere, upon the Plane of a Great Circle, by Lines drawn, from the Pole of the ſaid Circle to its Plane, thro' every Point of the Circles to be projected.

9. The Gnomonick Projection is the Repreſentation of the Circles of an Hemiſphere, upon a Plane parallel to that Great Circle which limits (or is the Baſe of) the Hemiſphere, by Lines drawn from the Center thereof, thro' all the Points of the Circle to be projected.

10. That Circle, on whoſe Plane the Projection is made, is called the Primitive Circle.

11. Thoſe Circles, whoſe Planes are parallel to the Primitive, are called Parallel Circles.

12. Thoſe Circles, whoſe Planes are perpendicular to the Primitive, are called Perpendicular Circles.

13. Thoſe Circles, whoſe Planes are inclined to the Primitive, are called Oblique Circles.

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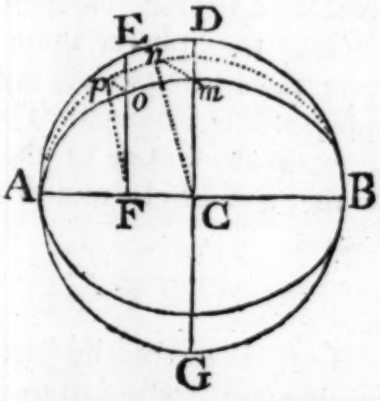
Of

ORTHOGRAPHICK PROJECTION.

PROPOSITION.

An Oblique Great Circle ApnB is projected into an Ellipſis, whoſe Conjugate Diameter is to its Tranſverſe, as the Coſine of the Angle, which that Circle makes with the Primitive, is to Radius.



FOR let AB be the Diameter of the Sphere, in which the propoſed Oblique Circle interſects the Primitive ADBG, and let CD, FE, C*n* and F*p* be perpendicular thereto, ſo that the Angle DC*n*, or EF*p*, may expreſs the Inclination of the two Planes to each other: Then, if *mn* and *po* be ſuppoſed perpendicular to CD and FE, the Projection of the Point *n* of the Oblique Circle will be in *m*, and that of *p* in *o* (by *Def.* 7.) But, becauſe of the ſimilar Triangles, *Cnm*, *Fpo*, it will be as *Cm*: *Cn* (CD) :: F*o*: F*p* (FE); therefore, as this Proportion every where holds, the Curve A*om*B paſſing thro' the Points *m*, *o* will be an Ellipſis; ſince, by a known Property of the Ellipſis, the Ordinates thereof are in a conſtant Ratio to the correſponding Ordinates of the circumſcribing Circle. Moreover, the Semi-Conjugate of the Ellipſis being C*m*, and the Angle C*mn* a Right Angle, it will be as (C*m*) the Semi-Conjugate, to (C*n*=AC) the Semi-Tranſverſe, ſo is the Coſine of (*m*C*n*) the Planes Inclination, to Radius. *Q. E. D.*

Corollary I.

Since it appears from the preceding Demonſtration that the Projection of any Oblique Circle, upon a Plane paſſing thro' its Center, is an Ellipſis; and becauſe the Lines, by which the Projection is made, are parallel to each other (*Def.* 7) it is evident that the Repreſentation of any Circle upon a Plane, parallel to the Primitive, will be the ſame as That upon the Primitive itſelf: Therefore, it follows, that the Repreſentation of any Leſſer Oblique Circle of the Sphere, upon the Primitive (as well as upon the Plane paſſing thro' its Center) is an Ellipſis, whoſe Conjugate Diameter is to its Tranſverſe, as the Coſine of the Angle which the Plane of the ſaid Circle makes with the Primitive is to Radius.

Corollary II.

If the Plane of the Circle A*pn*B be ſuppoſed to become perpendicular to That of the Primitive ADBG, the Ellipſis will degenerate into a Right Line; hence, it is manifeſt that any Circle, whoſe Plane is perpendicular to the Primitive, will be repreſented by a Right Line equal to its Diameter, and the Repreſentations of its Poles will be in the Extremities of that Diameter of the Primitive, which is perpendicular to the Repreſentation.

Corollary III.

It alſo follows, that the Arch (A*o*) of the Repreſentation of an Oblique Circle, cut off by a Leſſer Circle (FE) whoſe Plane is perpendicular to the Primitive, will be equal to the Arch (AE) of the Primitive cut off by the ſame Circle; and that the Arch of a Circle whoſe Plane is perpendicular to the Primitive (computed from the Center of the Repreſentation of that Circle) is projected into its Sine, becauſe

becauſe when the Ellipſis A*om*B degenerates to the Right Line AB, *n* coincides with C and *p* with F, and it is evident that CF is the Sine of ED, or its Equal *pn*. Whence it appears, that the Diſtance of any Point, in the Projection, from the Center of the Primitive, is equal to the Sine of the real Diſtance of that Point, from the Pole of the Primitive.

Corollary IV.

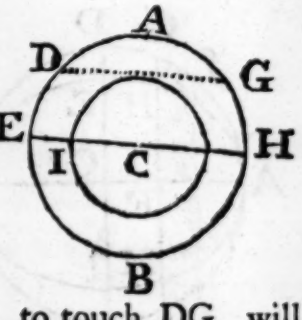
The Repreſentation of any Circle, parallel to the Primitive, is equal to the Circle itſelf and concentric with the Primitive; and the Repreſentation of its Pole, will be in the Center of the Primitive, by *Def.* 4.

Corollary V.

The Area of the Repreſentation of a Circle (or any other Plane) will be to the Area of the Circle (or Plane) itſelf, as the Coſine of the Angle which the given Plane makes with the Plane of Projection, is to Radius; for, F*o* being every where in Proportion to F*p* (FE) as C*m* to C*n* (CD), it is plain that the Sum of all the F*o*'s (or the Area A*om*B) will be to the Sum of all the FE's (or the Area ADB) in the ſame Ratio.

Problem I.

To deſcribe the Repreſentation of a Parallel Circle.



Let EH be a Diameter of the Primitive AHBE, and (*by Help of the Line of Chords*) make the Arch ED equal to the Diſtance of the given Parallel from its Pole (being That of the Primitive) draw DG parallel to EH; then a Circle deſcribed from the Center C, to touch DG, will be the Repreſentation required (*by Cor.* 4.)

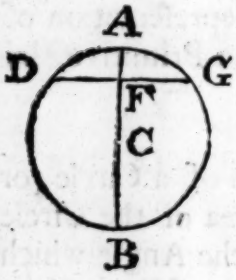
Otherwiſe,

Otherwiſe,

With the Radius CI, equal to the Sine of the Diſtance of the propoſed Parallel from the Pole of the Primitive, let a Circle be deſcribed, which will be the required Repreſentation (*by Cor.* 3.).

Problem II.

To deſcribe the Repreſentation of a Perpendicular Circle.



Let ADBG be the Primitive, and A the Pole of the given Circle; take AD and AG each equal to the Diſtance of the propoſed Circle from its Pole, and draw the Right Line DG which will be the Repreſentation required (*by Cor.* 2.)

Otherwiſe, from the Lines of Sines.

In the Diameter AB, ſet off CF equal to the Diſtance of the propoſed Circle from the Pole of the Primitive, and thro' F, perpendicular to AB, draw the Right Line DG for the required Repreſentation (*Cor.* 2 and 3.)

Problem III.

To deſcribe the Repreſentation of an Oblique Circle.

Case 1. Let the Circle propoſed be a Great one interſecting the Primitive ADBK in the Diameter A B, perpendicular to which draw the Diameter DK, and from the Points F, f, *&c.* in AB, taken at Pleaſure, draw the Perpendiculars FG, fg, *&c.*

In

In KD take CE (CI) to CD as the Coſine of the Angle of Inclination to Radius; and take FH (FM): FG: fh (fm): fg:: CE: CD; then a Curve deſcribed thro' all the Points A, h, H, E, B, I, M, m, *&c.* will be the Repreſentation required (*by the Prop.*) which (being an Ellipſis, whoſe Tranſverſe is AB and Conjugate ICE) may be otherwiſe deſcribed by the Conic Sections.

Note, The Diameter of the Primitive DK, perpendicular to the Interſection of the Oblique Circle with the Primitive, is called the Line of Meaſures both in This and the Stereographick Projection.

Case 2. Let the given Circle be a Leſſer one, and let AKBL be the Primitive; in the Diameter AB, from the Center C, ſet off (*by Cor.* 3.) CD and CE equal to the Sines of the Circle's neareſt and greateſt Diſtance from the Pole of the Primitive, reſpectively; biſect ED in F, and perpendicular to AB draw GFH, in which, from F, ſet off FG and FH each equal to the Sine of the Diſtance of the propoſed Circle from its Pole, then the Ellipſis EHDG deſcribed to the Tranſverſe HG and the Conjugate DE, will be the Repreſentation required (*by Cor.* 1.)

Problem IV.

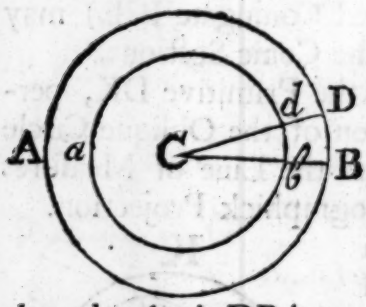
To determine the Repreſentation of the Poles of any Circle.

From the Center of the Primitive, ſet off (in the Line of Meaſures) the Sine of the Diſtance of the Pole of the given Circle from that of the Primitive.

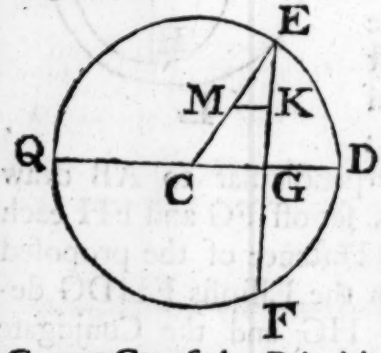
Problem

Problem V.

To determine the Number of Degrees, Minutes, &c. contained in any Part of the Repreſentation of a given Circle.



Case 1. Let the given Circle be a Parallel, whoſe Repreſentation is *adb*, and let *db* be the Part to be meaſured: Thro' *b* and *d*, from the Center of the Primitive, let the two Radii CB and CD be drawn, then the Arch DB intercepted between them (meaſured by Help of the Line of Chords) will ſhew the Number of Degrees, Minutes, *&c.* required.



Case 2. Let the given Circle be a Perpendicular one, whoſe Repreſentation is the Right Line EF, cutting the Diameter DQ, of the Primitive, which is perpendicular to it in G, and let KG be the Part to be meaſured; from the Center C, of the Primitive, draw CE, and parallel to QD draw KM, meeting CE in M, then CM (applied to the Line of Sines) will exhibit the Number of Degrees, Minutes, *&c.* required. For, ſince GK is the Sine of the Number of Degrees, *&c.* ſought, to the Radius EG, it is evident that CM will be the Sine of the ſame Number of Degrees, *&c.* to the Radius CE.

Case 3. Let the given Circle be an Oblique one, whoſe Repreſentation is EKGFH; and let the Arch GK intercepted between the Line of Meaſures

ſures AB and the Point K, be that which is to be meaſured; from O, the Center of the Ellipſis, thro' the Extremity E of the Tranſverſe Axis EF, let the Circle EMN be deſcribed, and thro' K parallel to AB let the Right-line LKM be drawn, meeting the Circle in M, then the Angle NOM will expreſs the required Number of Degrees, Minutes, *&c.* in the given Part.

From the Converſe of this laſt Problem, an Arch of a given Circle may be found to contain any propoſed Number of Degrees, Minutes, *&c.* alſo, from hence, the Repreſentation of a Circle, making a given Angle at a given Point with any Circle already projected, may be deſcribed.

The Application of the Orthographick *Projection to the Projection of the Circles of the* Celeſtial Sphere.

Definitions.

1. The Celeſtial Sphere is that in whoſe Surface the Sun, Moon, and Stars would, to the Eye of an Obſerver at the Earth's Center, appear to be placed.

If the Heavens be viewed from the Surface of the Earth, inſtead of the Center, the Appearance will not be ſenſibly different: For the Earth, compared with the Celeſtial Sphere, may be eſteemed as a Point.

2. The Axis of this Sphere is an imaginary Line, round which the Heavenly Bodies appear to revolve.

3. The Points, in which the Axis interſects the Surface of the Sphere, are called the Poles of the

World; one of them the North Pole, and the other the South Pole.

4. That Great Circle of the Celeſtial Sphere, which is equidiſtant from the two Poles of the World, is called the Equinoctial Circle.

5. Any Great Circle, paſſing thro' the Poles of the World, is called a Meridian, or an Hour Circle.

6. That Point in the Sphere, directly above the Head of a Spectator at any Place on the Surface of the Earth, is called the Zenith of that Place; and that Point, which is under his Feet, the Nadir.

7. That Great Circle, whoſe Poles are the Zenith and Nadir (in which the Surface of the Earth ſeems to interſect the Heavens) is called the Horizon.

8. Any Great Circle, paſſing thro' the Zenith, or Nadir, perpendicular to the Horizon, is called an Azimuth Circle.

9. That Great Circle, which paſſes thro' the Poles of the World, and the Zenith of any Place, is ſaid to be the Meridian of that Place; in which the Sun appears at Noon every Day thoughout the whole Year.

10. That Great Circle, which paſſes thro' the Zenith and Nadir perpendicular to the Meridian, is called the Prime Vertical.

11. That Circle, wherein the Sun appears to perform his Annual Revolution, is called the Ecliptic; which cuts the Equinoctial Circle in an Angle of 23° 29′.

12. Thoſe Points, in which the Equinoctial and Ecliptic interſect each other, are called the Equinoctial Points; being the Points in which the Sun appears when the Days and Nights are of equal Length.

13. Thoſe two Points in the Ecliptic, which are equidiſtant from the Equinoctial Points, are called the Solſtitial Points.

14. That Meridian, which paſſes thro' the Solſtitial Points of the Ecliptic, is called the Solſtitial Colure.

15. That Great Circle, which paſſes thro' the Equinoctial Points, and the Poles of the Ecliptic, is called the Equinoctial Colure *.

16. Thoſe Great Circles, which paſs thro' the Poles of the Ecliptic, are called Circles of Longitude.

17. Thoſe Leſſer Circles, whoſe Planes are parallel to the Plane of the Equinoctial, are called Parallels of Latitude or Declination; according as they belong to the Earth or Heavens.

18. Thoſe Leſſer Circles, whoſe Planes are parallel to the Plane of the Horizon, are called Parallels of Altitude.

19. Thoſe two Leſſer Circles, which are parallel to, and each 23° 29′ diſtant from the Equinoctial, are called Tropics (which limit the Sun's Progreſs to the North and South): That on the North Side the Tropic of Cancer, the other the Tropic of Capricorn.

20. Thoſe two Leſſer Circles, which are 23° 29′ diſtant from the Poles, are called Polar Circles; that next the North Pole the Arctic Circle, the other the Antarctic Circle.

* *The Ancients, and moſt of the Moderns, call that Circle the* EQUINOCTIAL COLURE *which paſſes thro' the Poles of the World, and the Equinoctial Points: But, as we do not perceive this Circle has any Uſe, in Geography or Aſtronomy, either in pointing out the Situation of Places, or the Poſition of the Stars, we have (conformable to ſome late Authors) choſe to call that Circle the* EQUINOCTIAL COLURE, *which is the firſt Circle of Longitude; from whence the Places of the Celeſtial Bodies are computed: The Propriety whereof we leave to the Judgment of the Critics.*

21. That Arch of an Azimuth Circle, which is included between the Horizon and the Center of the Sun or Star, *&c.* is ſaid to be the Altitude of the Sun, or Star, *&c.*

22. That Arch of the Horizon, intercepted between the Place of the Sun riſing, and the Eaſt Point of the Horizon, or between the Place of the Sun's Setting, and the Weſt Point of the Horizon, is called the Sun's Amplitude.

23. That Arch of the Horizon (or the Angle at the Zenith) included between the Meridian and the Azimuth Circle wherein the Sun appears, is called the Sun's Azimuth.

24. The Arch of the Equinoctial, intercepted between the Vernal Equinoctial Point, and the Meridian that the Sun or Star, *&c.* appears in, is called the Right Aſcenſion of the Sun or Star, *&c.*

25. The Arch of the Equinoctial, intercepted between the Vernal Equinoctial Point, and that Point of the Equinoctial which riſes with the Sun or Star, *&c.* is called the Oblique Aſcenſion of the Sun or Star, *&c.*

26. The Aſcenſional Difference is the Difference between the Right and Oblique Aſcenſions.

27. That Arch of the Ecliptic, intercepted between the Vernal Equinoctial Point, and the Circle of Longitude in which the Sun or Star, *&c.* appears, is ſaid to be the Longitude of the Sun or Star, *&c.*

28. The Arch of the Meridian, intercepted between the Sun, or a Star, and the Equinoctial, is called the Declination of the Sun or Star.

29. That Arch of a Circle of Longitude, intercepted between a Star and the Ecliptic, is ſaid to be the Latitude of the Star.

30.

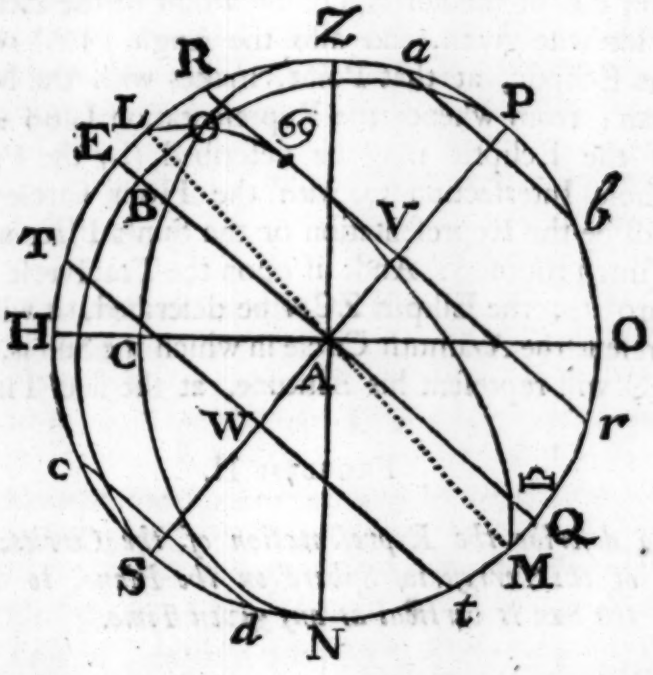
30. The Arch of the Meridian, intercepted between the Zenith of any Place and the Equinoctial, is called the Latitude of that Place.

31. That Arch of the Equinoctial (or the Angle at the Pole) included between the Meridians of any two Places on the Surface of the Earth, is ſaid to be the Difference of Longitudes of thoſe two Places.

32. The Arch of the Equinoctial (or the Angle at the Pole) included between the Meridian of the Place, and that Meridian where the Sun appears, is called the Hour Angle.

Problem I.

To deſcribe the Repreſentation of the Circles, &c. *of the Sphere on the Plane of the Meridian for the Latitude of* 51°. 32′. *North, on* May 20, *two Hours before Noon.*



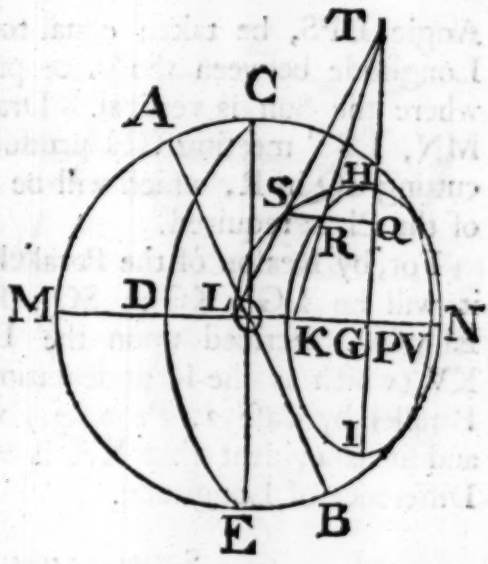
With the Chord of 60 Degrees, deſcribe the Circle HZON for the Primitive; draw the Diameters

ameters HO,ZN perpendicular to each other, the firſt to repreſent the Horizon, and the other the Prime Vertical, both which are Right Lines (by Cor. 2 of the Prop.) Make OP equal to the given Latitude, and draw the Diameter PS to repreſent the Hour Line of Six, perpendicular to which draw the Diameter EQ to repreſent the Equator. Take ET, ER, Q*t*, Q*r*, P*a*, P*b*, S*c* and S*d* each equal to 23°. 29′. and draw the Right Lines TW*t*, RV*r*, for the two Tropics, and *ab*, *cd* for the two Polar Circles. From A ſet off AB equal to the Sine of the given Hour from Six, and upon the Tranſverſe PS let the Ellipſis PBS be deſcribed. Moreover, if from the Sun's Right Aſcenſion the Hour from Noon be deducted, the Remainder will be the Right Aſcenſion of the culminating Point (or that Point E of the Equator which is in the Meridian) from which the Declination EL of the correſponding Point of the Ecliptic is likewiſe given, and alſo the Angle PLM which the Ecliptic, at that Point, makes with the Meridian; from whence the Repreſentation L 69 ♒M of the Ecliptic may be deſcribed (by the Prop.) whoſe Interſection ☉ with the Hour Circle PBS will be the Repreſentation of the Sun's Place at the Time propoſed: And, if upon the Tranſverſe ZN, thro' ☉, the Ellipſis ZCN be deſcribed, it will repreſent the Azimuth Circle in which the Sun is, and C☉ will repreſent his Altitude, at the ſaid Time.

Problem II.

To deſcribe the Repreſentation of the Circles, &c. of the Terreſtrial Sphere on the Plane, to which the Sun is vertical at any given Time.

Let MANE be the Primitive or Plane of Projection, MON the Repreſentation of the Meridian, which



which (paſſing thro' O the Repreſentation of the Place of the Sun in the Center of the Primitive) muſt be a Diameter; make the Angle NOB or MOA equal to the Angle which the Ecliptic makes with the Meridian paſſing thro' the given Place of the Sun, and draw the Diameter BOA for the Repreſentation of the Ecliptic. From the Line of Sines ſet off OD equal to the Sun's Declination, and on the Diameter CE perpendicular to MN, let the Ellipſis CDE be deſcribed for the Repreſentation of the Equator. Moreover, ſet off OP equal to the Coſine of the Sun's Declination, and P will be the Repreſentation of the Pole.

This Projection is of great Uſe in the Geometrical Conſtruction of Solar Eclipſes: Wherein the moſt material Point being to find the Repreſentation of ſome particular Place upon the Surface of the Earth; it may alſo be proper here to ſhew how the Repreſentation of any propoſed Place may be determined. In order to which, from the Line of Sines ſet off OK and OV equal to the neareſt and greateſt Diſtances of the Parallel of Latitude of the propoſed Place from the Point where the Sun is vertical; biſect KV with the Perpendicular IH in which take GI and GH each equal to the Sine of the Parallel's Diſtance from the Pole, with which as a Radius let the Quadrant HL be deſcribed interſecting MN in L, and let the Arch LS, or the Angle

Angle LPS, be taken equal to the Difference of Longitude between the Place propoſed, and That where the Sun is vertical. Draw SQ parallel to MN, LST meeting GH produced in T, and TK cutting SQ in R, which will be the Repreſentation of the Place required.

For, by Reaſon of the Parallel's LKG and SRQ, it will be LG : KG :: SQ : RQ; therefore an Ellipſis, deſcribed upon the Diameters IH and KV (which is the Repreſentation of the propoſed Parallel by Caſe 2. Prob. 3.) will paſs thro' R; and 'tis evident that KR is = LS = the given Difference of Longitude.

Scholium.

This Kind of Projection (as all Oblique Circles are repreſented by Ellipſes, which are difficult to deſcribe) has been but ſlightly treated of by moſt Authors on the Subject; yet it's Uſe in the Conſtruction of Spherical Problems (the chief Purpoſe and End of all Projections) is not inferior to that of the Stereographick Projection, but has the Advantage of it in almoſt all Reſpects: Since, by a proper Application of the foregoing Propoſitions, every Thing may be performed by the Means of Right-Lines, as will abundantly appear from the following Example: *Wherein the Sun's Amplitude, the Time of his Riſing and Setting, and alſo his Azimuth and Altitude on the Day, and at the Hour of the Day, ſpecified in the firſt Problem, are ſeverally required.*

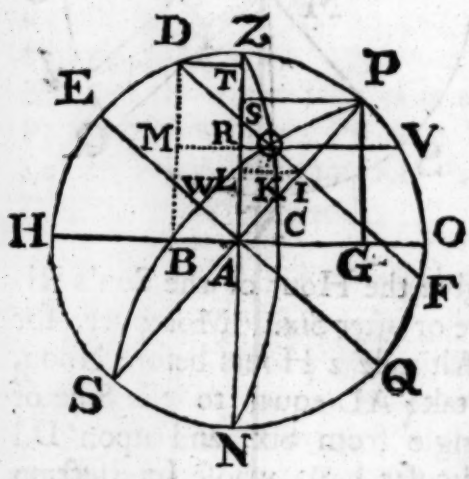
Having deſcribed, upon the Plane of the Meridian HZON, the Repreſentation HO of the Horizon, ZN that of the Prime Vertical, PS that of the Hour Line of Six, and EQ that of the Equator, as in the former of the preceding Problems; ſet off AF equal to the Sine of the Sun's Declination, and parallel to EQ, draw Dd for the Repreſentation of the Sun's Parallel

Parallel of Declination (*by Cor. 2 of the Prop.*) cutting HO in B, then AB upon the Line of Sines will give the required Amplitude. And if A, *d* be joined and BK be drawn, perpendicular to Dd, meeting Ad in K, then AK upon the Sines will give the Hour of the Sun's Rising or Setting before or after Six. Moreover, for the Azimuth and Altitude 2 Hours before Noon, join A, D in which take AL equal to the Sine of (60°.) the Hour Angle from Six; and upon Dd let fall the Perpendicular L⊙, whose Intersection with Dd will be the true Representation of the Place of the Sun in the Projection: And therefore ⊙M, perpendicular to HO, on the Line of Sines will give the Sun's Altitude at the proposed Time. Furthermore, if the Representation G⊙g of the Parallel of Altitude be drawn, and also AG intersecting ⊙M in I, then AI on the Line of Sines will give the Sun's Azimuth at the same Time.

Besides the Advantage the Orthographick Projection has above the Stereographick, in the Geometrical Construction of Spherical Problems, there are several useful Theorems which may be derived from the Former, that by the Latter are altogether impracticable: Whereof I shall put down two or three of the most considerable.

THEOREM I.

The Rectangle of the Sines of any two Sides of a Spherical Triangle, drawn into the Verſed Sine of the Angle they include, being added to the Verſed Sine of the Difference of the ſaid Sides, will be equal to the Verſed Sine of the third Side.



For let HZON be a Projection of the Sphere on the Plane of a great Circle, whereof ZP is a Part; and let PAS and EAQ be two Diameters, the former paſſing thro' the Angular Point P and the other perpendicular thereto; alſo let ZAN and HAO be two other Diameters whereof the former paſſes through the Angular Point Z and is perpendicular to the latter. Let DF be drawn through the other Angular Point ⊙ parallel to EQ, and let the Perpendiculars PG, PS, DB, DT, V⊙M, IL and ⊙K be drawn; then AG (PS) will be the Sine and PG the Coſine of ZP, DI the Sine and AI the Coſine of P⊙ or PD, AB (DT) the Sine and DB (AT) the Coſine of DZ, EW the Verſed Sine and AW the Coſine of the Angle P, VR the Sine and AR the Coſine of Z⊙, and ZT the Verſed Sine of DZ. Then, by ſimilar Triangles, AE (AP) : AG : : D⊙ : DM; and by the Property of the Ellipſis AE : DI : : EW : D⊙; whence by compounding the two Proportions $AE^2 : AG \times DI :: EW : DM = TR$; $= \frac{AG \times DI \times EW}{AE^2}$, therefore $\frac{AG \times DI \times EW}{AE^2}$ +

$+TZ=RZ$, which, when AE is equal to Unity, becomes $AG \times DI \times EW + TZ$. *W. W. D.*

Theorem II.

From the Verſed Sine of the Side oppoſite the required Angle, take the Verſed Sine of the Difference of the other two Sides, divide the Remainder by the Rectangle of the Sines of thoſe two Sides, and the Quotient will be the Verſed Sine of the Angle ſought.

For, by the laſt $AG \times DI \times EW + TZ = RZ$, therefore $\frac{RZ - TZ}{AG \times DI} = EW$. *W. W. D.*

Theorem III.

The Rectangle of the Sines of any two Sides of a Spherical Triangle, drawn into the Coſine of the Angle they include, being added to the Rectangle of their Coſines, will be equal to the Coſine of the third Side.

For AE : AW :: DI : ⊙I (by the Property of Ellipſis) alſo by ſimilar Triangles AE : PG :: AI : AL and AE : AG :: ⊙I : ⊙K = RL, therefore $AR = AL + RL = \frac{AI \times PG}{AE} + \frac{DI \times AW}{AE} \times \frac{AG}{AE}$; which, by ſuppoſing AE equal to Unity, becomes $AI \times PG + DI \times AW \times AG$. *W. W. D.*

Theorem IV.

The Rectangle of the Sines of any two Sides of a Spherical Triangle, drawn into the Verſed Sine of the Angle they include, being taken from the

Coſine of the Difference of the ſaid Sides, will be equal to the Coſine of the third Side.

For AE : EW :: DI : D⊙ = $\frac{EW \times DI}{AE}$ and AE : AG :: D⊙ : DM = $\frac{AG \times EW \times DI}{AE^2}$; therefore, AE being equal to Unity, AR = DB — AG × EW × DI. *W. W. D.*

From the different Values of AR, in the two laſt Theorems, we may derive two Expreſſions, *viz.* AW = $\frac{AR - PG \times AI}{AG \times DI}$ and EW = $\frac{DB - AR}{AG \times DI}$, which, put in Words, gives the two following Theorems for determining an Angle when all the Sides are given.

Theorem V.

From the Coſine of the Side oppoſite the required Angle, take the Rectangle of the Coſines of the Sides including it; divide the Remainder by the Rectangle of the Sines of thoſe Sides, and the Quotient will be the Coſine of the required Angle.

Theorem VI.

From the Coſine of the Difference of the Sides including the required Angle, take the Coſine of the Side oppoſite; divide the Remainder by the Rectangle of the Sines of the including Sides, the Quotient will be the Verſed Sine of the required Angle.

The Uſe of theſe laſt Theorems will eaſily appear to thoſe, who have been much converſant in the Application of Algebra to the Reſolution of the more difficult Caſes of Spherical Triangles.

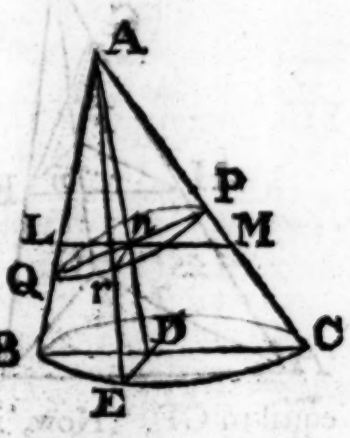
OF

OF THE

STEREOGRAPHICK PROJECTION.

LEMMA I.

If an Oblique Cone, having a circular Baſe BFCE, be cut by a Plane QmPr ſo that the Angle APQ may be equal to ABC, where the Triangle ABC is a Section of the Cone by a Plane perpendicular to the Baſe through the Diameter BC; I ſay the Section QmPr will be a Circle.

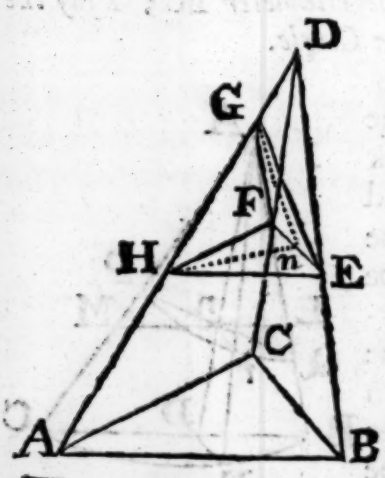


FOR let the Right Line ED be perpendicular to BC cutting it in D, and from the Vertex A of the Cone let AE and AD be drawn, the former interſecting the Periphery of the propoſed Section Q*m* P*r* in *r*, and the latter the Line QP in *n*; let L*n*M be drawn parallel to BC, and join *r*, *n*: Then, by Reaſon of the ſimilar Triangles ABD, AL*n*; ADC, A*n*M; and L*n*Q, P*n*M, it will be as CD : M*n* :: (AD : A*n* ::) DE : *rn*, and as BD : L*n* :: (AD : A*n* ::) DE : *rn*, therefore BD × CD : L*n* × M*n* :: $\overline{DE}^2$: $\overline{rn}^2$. But by the Property of the Circle $\overline{DE}^2$ = BD × CD, therefore $\overline{rn}^2$ = L*n* × M*n*. Moreover, from the ſimilar Triangles L*n*Q, P*n*M, it will be L*n* : Q*n* : : P*n* : M*n*, therefore L*n* × M*n* = Q*n* × P*n* = $\overline{rn}^2$, which is a known Property of the Circle.

LEMMA

Lemma II.

If an Oblique Pyramid ABCD, whoſe Baſe is the Iſoſceles Triangle ABC, whereof the Extremities B, C of the equal Sides AB, AC are equally remote from the Vertex D, be cut by a Plane EFG, ſo that its Interſection FE with the Plane BCD be parallel to BC, and its Inclination to the Line AD be equal to the Inclination of the Baſe to the ſaid Line; I ſay the Triangle EFG will be ſimilar to BAC.



Let the Plane FHE be parallel to the Baſe CAB, then it is too plain to need a Demonſtration here, that the Triangles FHE and CAB are ſimilar as well as DFE and DCB: Whence, DF and DE being equal, the Angle FDG = the Angle EDG, and DG common, it is plain that GF will be equal to GE. Now, if H*n* and G*n*, be drawn to meet in the Middle of FE, it will appear that the Angle DH*n* will expreſs the Inclination of the Plane FHE to the Line AD, and HG*n* that of the other Plane FGE; therefore, theſe Angles being equal, H*n* will be equal to G*n* and therefore the Iſoſceles Triangles EHF and EGF having the ſame Baſe and equal Altitudes are equal in all Reſpects, and ſo the Angle EGF = EHF = BAC. *Q. E. D.*

Corollary I.

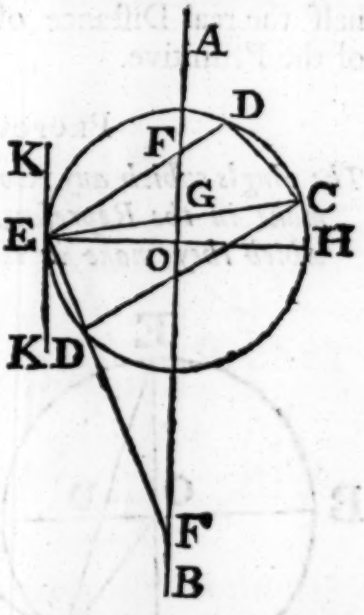
Hence, if a right-lined Angle BAC be projected upon a Plane FGE, inclined to the Line

D

DA paſſing through the Eye D and the Angular Point A, in an Angle equal to the Inclination of the Plane of the given Angle, but contrariwiſe poſited, then the Angle in the Repreſentation will be equal to the Angle given.

Proposition I.

Every Circle, whoſe Plane does not paſs through the Eye, has a Circle for its Repreſentation.



Let E be the Place of the Eye, AB the Plane of Projection perpendicular to EO, and infinitely extended every Way, and let CD be the Diameter of the Circle to be projected, and GF its Repreſentation; alſo, ſuppoſe Rays from every Point of the Circle, whoſe Diameter is CD, and whoſe Plane is perpendicular to that of the Circle ECD, to come to the Eye at E: Then, it is manifeſt they will form an Oblique Cone, the Diameter of whoſe Baſe is CD; but GF (AB) the Plane of Projection makes the Angle G equal to the Angle D, and conſequently the Angle F equal to the Angle C: Becauſe, if EK be ſuppoſed parallel to AB, the Angle KEG or its Equal FGE will be equal to D (by *Eu.* 3.32;) therefore the Repreſentation of the Circle whoſe Diameter is CD will alſo be a Circle (by Lemma 1.) *Q. E. D.*

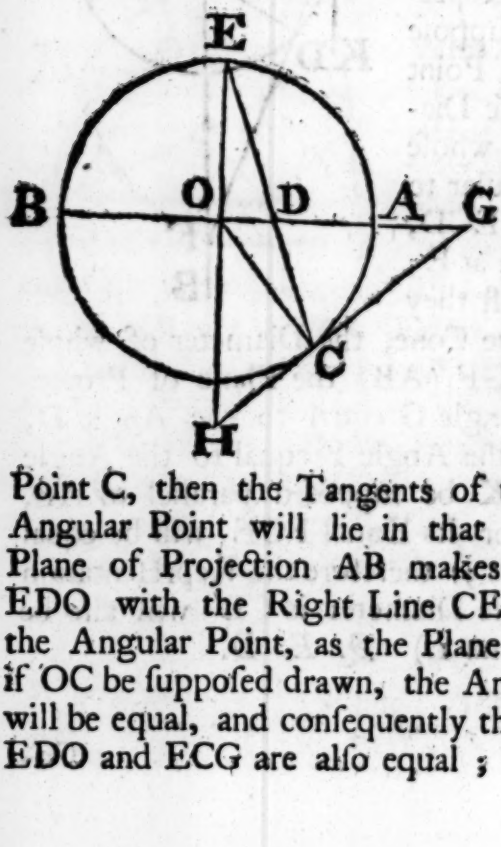
Co-

COROLLARY.

Becauſe O G is the Repreſentation of the Arch CH and equal to the Tangent of OEG, or half that Arch, it follows, that an Arch of a Great Circle, whoſe Plane ECD paſſes through the Eye, will be projected into the Tangent of half that Arch; and conſequently, that the Repreſentation of any Point in a Sphere will be found by ſetting off from the Center of the Primitive, the Tangent of half the real Diſtance of that Point from the Pole of the Primitive.

PROPOSITION II.

The Angle which any two Circles, or their Tangents, make in the Repreſentation is equal to the Angle which they make in the Sphere.



For let E be the Eye, AB the Plane of Projection, and C the Angular Point where the Peripheries of the two Circles interſect each other; and ſuppoſe GH to be a Plane touching the Sphere in the Point C, then the Tangents of the Circles at the Angular Point will lie in that Plane: But the Plane of Projection AB makes the ſame Angle EDO with the Right Line CE, paſſing through the Angular Point, as the Plane GH doth; for, if OC be ſuppoſed drawn, the Angles E and OCE will be equal, and conſequently their Complements EDO and ECG are alſo equal; from whence and

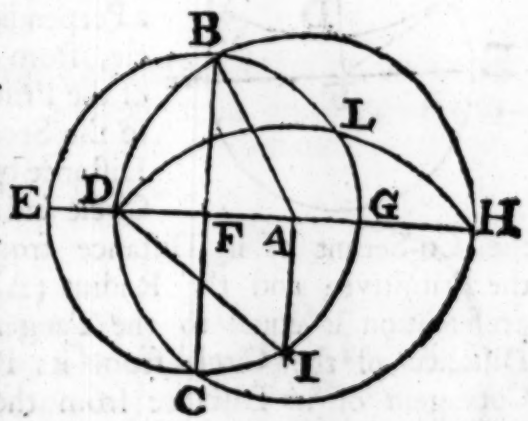
from

from Lemma 2, the Truth of the Propoſition is manifeſt.

Corollary I.

Since the Angle made by any two Circles, at the Point of Interſection, is equal to the Angle made by their Radii, drawn to the ſame Point; it follows that the Angle made by the Radii, drawn to the Interſection of the Repreſentations of two Circles, will be equal to the real Angle made by the Circles on the Sphere.

Corollary II.



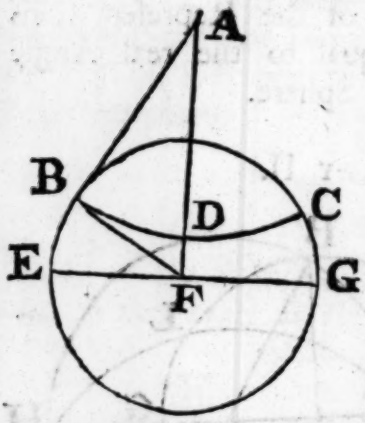
Hence the Diſtance of (A) the Center of the Repreſentation of a Great Circle, from (F) the Center of the Primitive, is equal to the Tangent of the Inclination of the Plane of the ſaid Circle to the Plane of the Primitive; and the Radius (AB) of the Repreſentation is equal to the Secant of that Inclination.

Corollary III.

Moreover, it follows that the Center I of the Repreſentation of any other Great Circle paſſing through D will be in the Right Line AI, drawn from A, the Center of the former, perpendicular to the Diameter EG. For, ſince all Great Circles cut each other in oppoſite Points of the Sphere, it is plain, that the propoſed Repreſentation DL, will paſs through H the Projection of the Point

oppoſite to D; and therefore its Center, being equally diſtant from H and D, muſt be in the Perpendicular AI. Hence, it appears that the Radius of the Circle DLH will be to That of CDB as the Secant of the Inclination of the two Circles to Radius.

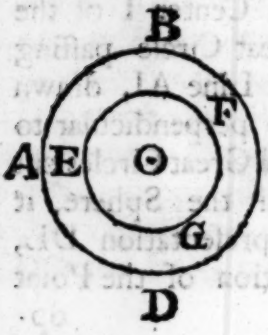
COROLLARY IV.



It alſo appears that the Diſtance of (A) the Center of the Repreſentation of any Leſſer Circle, whoſe Plane is parallel to That of a Perpendicular Great Circle, from (F) the Center of the Primitive, is equal to the Secant of the real Diſtance between the ſaid Circle and its Pole, or to the Co-Secant of its Diſtance from the Pole of the Primitive; and the Radius (AB) of the Repreſentation is equal to the Tangent of the real Diſtance of the Circle from its Pole, or to the Cotangent of its Diſtance from the Pole of the Primitive.

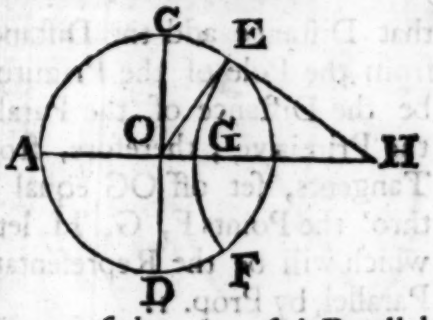
PROBLEM I.

To deſcribe the Repreſentation of any Leſſer Circle of the Sphere, which is parallel to a given Great Circle.



CASE 1. Let the Great Circle given be the Primitive; from the Center O, with an Interval OE equal to the Diſtance of the propoſed Parallel from its Pole, deſcribe the Circle EFG for the Repreſentation required by Cor. to Prop. 1.

CASE



Case 2. Let the given Circle be a Perpendicular Circle, whoſe Repreſentation is CD; from the Line of Semi-Tangents ſet off OG, perpendicular to CD and equal to the Diſtance of the propoſed Parallel from the given Circle, in which produced take GH equal to the Tangent Complement of the ſame Diſtance; then, with the Interval HG, deſcribe EGF for the required Repreſentation, by Cor. to Prop. 1. and Cor. 4. Prop. 2.

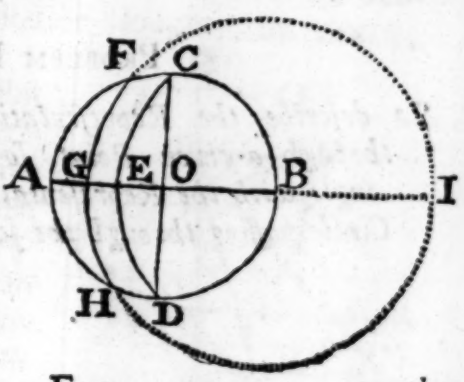
Otherwiſe,

Set off OH equal to the Co-ſecant of the Parallels Diſtance from the Great Circle, and take HG equal to the Cotangent of the ſame Diſtance, by Prop. 2. Cor. 4. then proceed as above.

Otherwiſe.

Make CE equal to the Diſtance of the Parallel from the great Circle, draw OE, and EH perpendicular thereto with which, as Radius, deſcribe EF as before; which (by Cor. 4. Prop. 2.) will be the Repreſentation required.

Case 3. Let the Circle given be an Oblique One, whoſe Repreſentation is DEC; ſet off CF and DH each equal to the Diſtance of the propoſed Parallel from the ſaid Circle; alſo, to



that Diſtance add the Diſtance of the given Circle from the Pole of the Primitive, and the Sum will be the Diſtance of the Parallel from the Pole of the Primitive; therefore, from the Line of Semi-Tangents, ſet off OG equal to that Diſtance, and thro' the Points F, G, H let a Circle be deſcribed, which will be the Repreſentation of the required Parallel, by Prop. 1.

Otherwiſe,

Having found (OG) the neareſt Diſtance of the Parallel from the Center of the Primitive, ſubſtract it from twice the Diſtance of the Parallel from its Pole, and from the Line of Semi-Tangents ſet off OI equal to the Remainder; then upon GI, as a Diameter, deſcribe the Circle GFIH, as before, which will be the Repreſentation required, by Cor. Prop. 1.

Problem II.

To determine the Repreſentation of the Pole of any Circle.

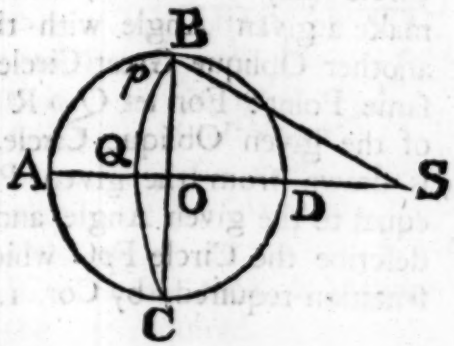
From the Center of the Primitive, ſet off (in the Line of Meaſures) the Semi-Tangent of the Diſtance of the Pole of the given Circle from That of the Primitive.

Problem III.

To deſcribe the Repreſentation of a Great Circle through a given Point, ſo as to make a given Angle with the Repreſentation of a Perpendicular Circle paſſing through the ſame Point.

Case

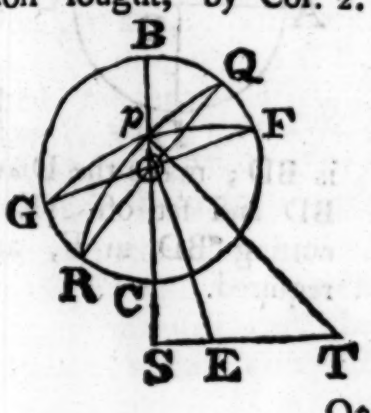
Case 1. Let BC be the Repreſentation of the propoſed Circle, and let the given Point (*p*) be in the Center of the Primitive; from the Line of Chords ſet off BQ equal to the given Angle and draw QpR, then BpQ will be the Angle ſought by Prop. 2. Cor. 1.



Case 2. Let the Point (*p*) be in the Periphery of the Primitive, and let BC be the Repreſentation of the given Circle, to which AD is perpendicuticular; make the Angle OBS equal to the Complement of the given Angle, and upon S, with the Radius BS, deſcribe the Circle BQC, which will be the required Repreſentation, by Cor. 1. Prop. 2.

Otherwiſe,

From the Line of Secants, ſet off *p*S (equal to the Complement of the given Angle and meeting AD produced in S) which will be the Radius of the Repreſentation ſought, by Cor. 2. Prop. 2.



Case 3. Let the Point (*p*) between the Periphery of the Primitive and its Center, at a given Diſtance in the Repreſentation BC of the given Circle; from the Center, in BC, ſet off OS equal to the Cotangent of

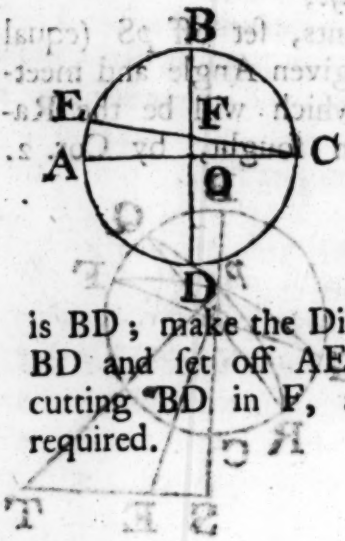
O*p*, and make ST perpendicular to BC and the Angle S*p*T equal to the Complement of the given Angle; then upon the Center T, with the Radius T*p*, deſcribe Q*p*R for the Repreſentation required, by Cor. 1. and 3. of Prop. 2.

Corollary.

Hence, the Repreſentation of any Oblique Great Circle may be drawn, through a given Point, to make a given Angle with the Repreſentation of another Oblique Great Circle paſſing through the ſame Point: For let Q*p*R be the Repreſentation of the given Oblique Circle, whoſe Radius T*p* is drawn from the given Point *p*; make T*p*E equal to the given Angle and with E*p*, as Radius, deſcribe the Circle F*p*G which will be the Repreſentation required, by Cor. 1. and 3. Prop. 2.

Problem IV.

To cut a given Arch from the Repreſentation of any Great Circle.



Case 1. Let the given Circle be the Primitive; then, from the Line of Chords, ſet off AE equal to the propoſed Arch.

Case 2. Let the Circle propoſed be a Perpendicular Circle, whoſe Repreſentation is BD; make the Diameter AC Perpendicular to BD and ſet off AE as before; then draw EC cutting BD in F, and QF will be the Arch required.

Otherwiſe,

Otherwiſe,

In OB, ſet off, from the Line of Semi-Tangents, OF equal to the given Arc, by Cor. Prop. 1.

CASE 3. Let the Circle propoſed be an Oblique Great Circle, whoſe Repreſentation is BID; let AC be the Repreſentation of another Great Circle, whoſe Plane is perpendicular to the Primitive and to the given Circle, then at a Diſtance from AC, which is equal to the Arc to be cut off, deſcribe the Parallel EF (by Caſe 2. Prob. 1.) cutting BID in G; then, it is manifeſt that, IG will be the Arc required.

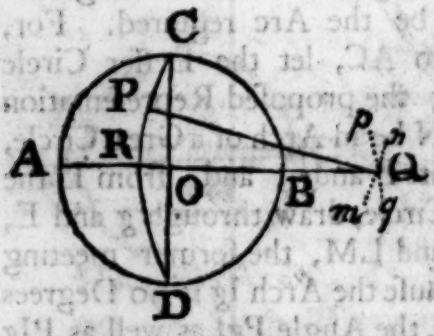
Otherwiſe,

Find (P) the Repreſentation of the Pole of the given Circle, and ſet off AE equal to the given Arc, then draw the Right Line EP cutting BID in G, and IG will be the Arc required. For, through E, parallel to AC, let the Leſſer Circle EF be drawn, cutting the propoſed Repreſentation BID in *g*; alſo let P*g*N be an Arch of a Great Circle, paſſing thro' the Points P and *g*; and, from L the Center of the Leſſer Circle, draw through *g* and E, the Right Lines LK and LM, the former meeting CA in K; then, becauſe the Arch I*g* is 90 Degrees diſtant from its Pole P, the Angle P*g*I as well as PI*g* will be a Right Angle. Therefore, ſince the Angles in the Projection are the ſame as in the Sphere itſelf, the two Circles F*g*N and E*g*F will touch each other in the Point *g*, it being manifeſt that the Angle B*g*H, which the Parallel makes with the Great Circle BID is a Right Angle. Hence, it fol-

follows that the Center of the Great Circle PgN muſt fall (at M) ſomewhere in the Right Line Lg. Join M, P which will be parallel to LK; for AE, or its Equal Ig, is the Meaſure of the mixed lined Angle IPg, whoſe Complement is the Angle MPA. But AE is the Meaſure of the Angle AOE, whoſe Complement is the Angle K; therefore the Angles MPA and K being equal, becauſe they are both Complements to AE, the Lines PM and LK are Parallels as was to be ſhewn. Whence, it alſo appears that the Angles PMg and ELg will likewiſe be equal: Therefore, if two Right Lines Eg and gP be drawn, then becauſe the Angles at the Vertices of the Iſoſceles Triangles PMg and ELg are equal, the Angles at the Baſe MPg and LgE are alſo equal, conſequently EgP one Right Line. *Q. E. D.*

Problem V.

To draw the Repreſentation of a Great Circle thro' any given Point (P) to make a given Angle with the Primitive.

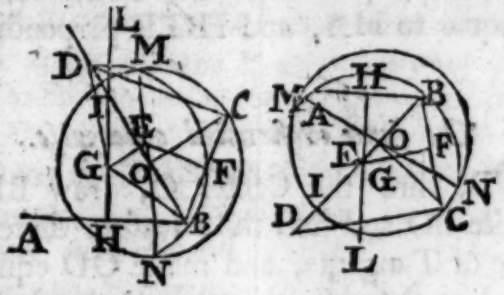


Upon P, as a Center, with a Radius equal to the Secant of the given Angle, deſcribe the Arc *mn*; alſo, upon O, the Center of the Primitive, with a Radius equal to the Tangent of the ſaid Angle, deſcribe the Arc *pq*, cutting the former in Q; then about Q, as a Center, through P deſcribe the Circle CPD, which will be the Repreſentation ſought, by Cor. 2. Prop. 2.

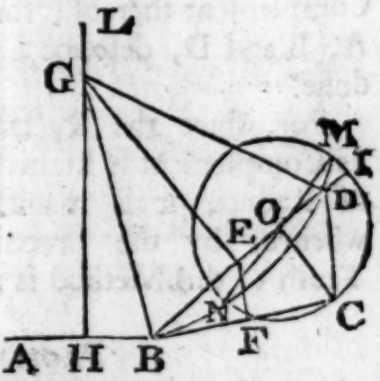
Pro-

Problem VI.

To deſcribe the Repreſentation of a Great Circle through two given Points, A and B.



Draw AB, which biſect with the Perpendicular HL; from (B) one of the Points, thro' (O) the Center of the Primitive draw BI, perpendicular to which draw OC meeting the Primitive in C. Join B, C which biſect with the Perpendicular EF, meeting BO in E; and, perpendicular to BO, draw EG meeting HL in G; then, if with the Radius G B a Circle be deſcribed, the Thing is done.



Demonstration.

Draw CD perpendicular to BC meeting BI in D, and join O, M and O, N; then, becauſe BF is equal to FC and DC parallel to EF, it follows that DE is equal to EB; whence, becauſe GE is perpendicular to DB, GD and GB will alſo be equal; conſequently the Circle paſſes through the Point D. Moreover, by Reaſon of the ſimilar Triangles BCO and CDO, we have BO : OC :: OC : OD; therefore BO × OD (= OC^2) = OM × ON; whence, it follows that,

MON muſt be a Right Line, and conſequently BNDM, cutting the Periphery of the Primitive in two equal Parts, muſt be a Great Circle; which Circle will alſo paſs thro' the Point A, becauſe HB is equal to HA, and HG is perpendicular to AB.

The ſame conſtructed otherwiſe.

From B, thro' the Center O, draw BI as before; take BO and find the Meaſure thereof upon the Line of Tangents, and make OD equal to the Complement thereof; then, thro' the three Points A, B and D, deſcribe a Circle and the Thing is done.

For, ſince the Rectangle of any Tangent and its Complement is known to be equal to the Square of Radius, it is manifeſt that $BO \times OD = OI^2$; whence, by the preceding Demonſtration, the Truth of this Method is alſo manifeſt.

Problem VII.

The Repreſentation of the Pole of any Circle being given; to deſcribe the Repreſentation of the Circle itſelf.

Let Q be the Repreſentation of the Pole, and from E the Place of the Eye, thro' Q, draw EQP meeting the Periphery of the Circle EAH in P; from the Line of Chords ſet off PD=PC= the given Diſtance of the Circle from its Pole; draw ED and EC interſecting AB in F and G; then FG will be the Diameter of the Circle required, by Prop. 1.

Problem

Problem VIII.

The Repreſentation of a given Point, and alſo of the Pole of a Circle paſſing thro' that Point, being given; to deſcribe the Repreſentation of the Circle itſelf.

Let D be the Repreſentation of the given Point, and B that of the Pole, thro' B and C (the Center of the Primitive) draw the Right line CBH, and thro' D and B (by Prob. 6) deſcribe the Great Circle EDBF whoſe Center is G; join G, D perpendicular to which draw DA, meeting CH in A, then DA will be the Radius of the Repreſentation required, by Prop. 2. Cor. 1. and 4.

Problem IX.

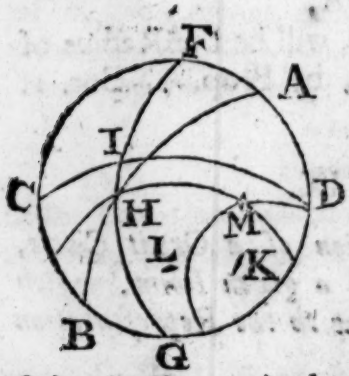
To deſcribe the Repreſentation of a Great Circle, thro' the Repreſentation of a given Point, which ſhall make a given Angle with the Repreſentation of a given Great Circle.

Let ANB be the Repreſentation of the given Great Circle, and D that of the given Point; about D, as the Repreſentation of a Pole, let HG the Repreſentation of a Great Circle be deſcribed; find E the Repreſentation of the Pole of the Circle whereof ANB is the Repreſentation, at a Diſtance from which equal to That of the given Angle draw the Parallel IFK interſecting HG in

F; about F, as the Repreſentation of a Pole, deſcribe the Repreſentation of a Great Circle, which will paſs thro' the Point D, and interſect ANB in the Point N, forming an Angle equal to That given. For, ſince the Angle formed at the Interſection of two Great Circles on the Sphere is equal to the Arch included between their Poles, it is evident from Cor. 1. Prop. 2. that the Angle ANL will be equal to the given Angle.

Problem X.

To deſcribe the Repreſentation of a Great Circle, which ſhall interſect the Repreſentations of two other given Great Circles in given Angles.



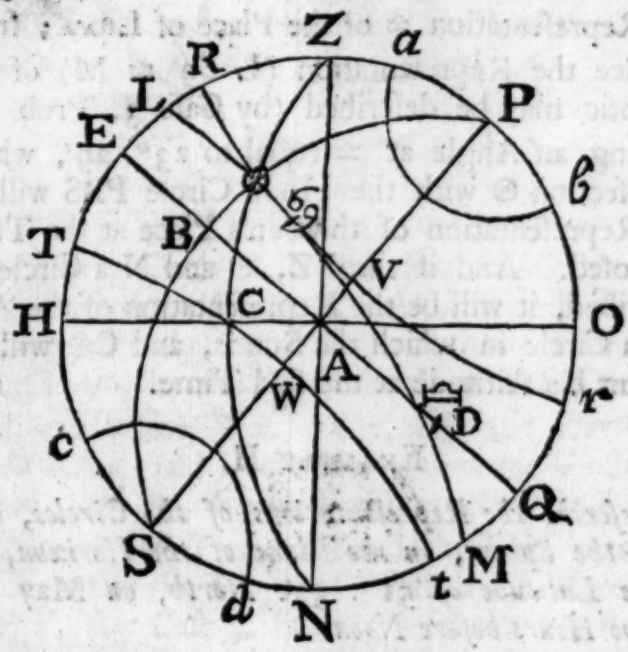
Let AB and CD be the Repreſentations of the given Circles, and let K and L the Repreſentations of their Poles be found; about each of which let the Repreſentations of two Parallels be deſcribed, reſpectively, equal to the Diſtance of the Angle required to be formed with the Circle correſponding to the reſpective Poles; then the Point M, where theſe two Parallels interſect each other, will be the Repreſentation of the Pole of the Circle whoſe Repreſentation is required; the Demonſtration whereof is evident from the ſame Conſideration as That in the preceding Problem.

The

The APPLICATION *of the* STEREOGRAPHICK PROJECTION, *to the Projection of the Circles of the* CELESTIAL SPHERE.

EXAMPLE I.

To deſcribe the Repreſentation of the Circles, &c. of the Sphere, on the Plane of the Meridian, for the Latitude of 51°. 32'. *North, on* May 20, *two Hours before Noon.*



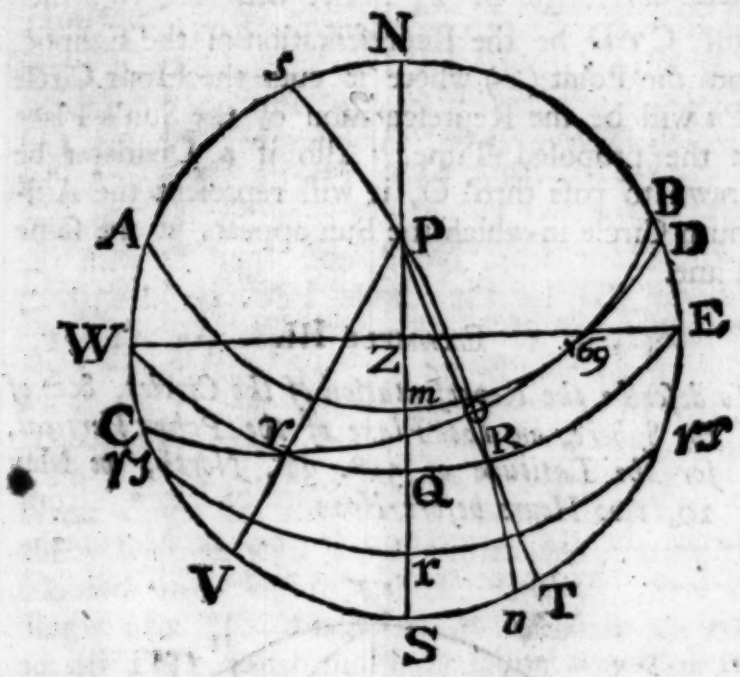
With the Chord of 60 Degrees deſcribe the Circle HZON for the Primitive; draw the Diameters HO, ZN perpendicular to each other, the firſt to repreſent the Horizon, and the other the Prime Vertical; both which are Right-Lines (by Prop. 1.) Make OP equal to the given Latitude, and draw the Diameter PS to repreſent the Hour Line of Six, perpendicular to which draw the

the Diameter EQ to repreſent the Equator. Take ET, ER, Q*t*, Q*r*, P*a*, P*b*, S*c* and S*d* each equal to 23°. 29′. and (by Caſe 2. Prob. 1.) draw the Parallels TW*t*, RV*r*, for the two Tropics and *ab*, *cd* for the two Polar Circles. From A, by Caſe 2. Prob. 3. ſet off AD equal to the Tangent of the Hour from Noon, and upon D with the Interval PD let the Circle PBS be deſcribed. Moreover, if to the Sun's Right Aſcenſion the Hour from Six be added, and their Sum be deducted from 180 Degrees, the Semi-Tangent of that Remainder ſet off from A towards Q will give the Repreſentation ♎ of the Place of Libra; from whence the Repreſentation (L 69 ♎ M) of the Ecliptic may be deſcribed (by Caſe 3. Prob. 3.) making an Angle at ♎ equal to 23°. 29′, whoſe Interſection ⊙ with the Hour Circle PBS will be the Repreſentation of the Sun's Place at the Time propoſed. And if thro' Z, ⊙ and N a Circle be deſcribed, it will be the Repreſentation of the Azimuth Circle in which the Sun is, and C⊙ will repreſent his Altitude at the ſaid Time.

Example II.

To deſcribe the Repreſentations of the Circles, &c. *of the Sphere, on the Plane of the Horizon, for the Latitude of* 51°. 32′. *North, on* May 20, *two Hours before Noon.*

With the Chord of ſixty Degrees deſcribe the Circle SWNE for the Primitive; draw the Diameters NS, WE perpendicular to each other, the firſt to repreſent the Meridian, and the other the Prime Vertical. From the Center Z, repreſenting the Zenith, ſet off from the Line of Semi-Tangents ZQ equal to the Latitude, and ZP equal to the Complement thereof; then will P repreſent the Pole



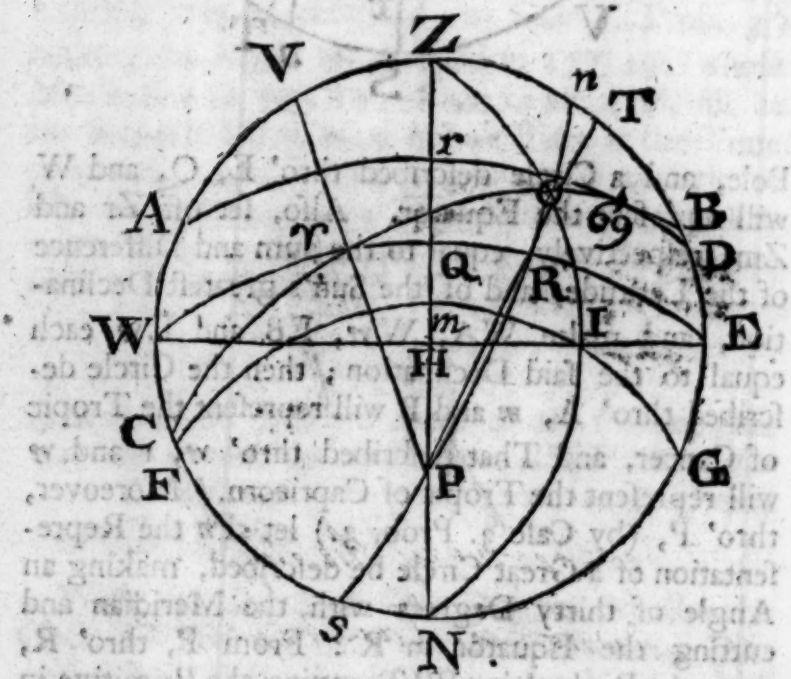
Pole, and a Circle deſcribed thro' E, Q, and W will repreſent the Equator. Alſo, ſet off Zr and Zm, reſpectively, equal to the Sum and Difference of the Latitude, and of the Sun's greateſt Declination, and make WA, W♈, EB and E♈ each equal to the ſaid Declination; then the Circle deſcribed thro' A, *m* and B will repreſent the Tropic of Cancer, and That deſcribed thro' ♈, r and ♈ will repreſent the Tropic of Capricorn. Moreover, thro' P, (by Caſe 3. Prob. 3.) let *s*P*n* the Repreſentation of a Great Circle be deſcribed, making an Angle of thirty Degrees with the Meridian and cutting the Equator in R: From P, thro' R, draw the Right-Line PRT cutting the Primitive in T; then ſet off TV, equal to the Sun's Right Aſcenſion, and draw the Right Line PV cutting the Equator in ♈: Furthermore, thro' ♈ deſcribe (C♈D) the Repreſentation of a Great Circle, to

and ſet the contrary way the make
difference between Zr, Zm & the diſſ.
of the Tropics from the N. pole.

make an Angle of 23°. 29′. with EQW; then will C♈D be the Repreſentation of the Ecliptic, and the Point (☉) where it cuts the Hour Circle *s*P*n* will be the Repreſentation of the Sun's Place at the propoſed Time. Alſo if a Diameter be drawn to paſs thro' ☉, it will repreſent the Azimuth Circle in which the Sun appears at the ſame Time.

Example III.

To deſcribe the Repreſentation of the Circles, &c. *of the Sphere, on the Plane of the Prime Vertical, for the Latitude of* 51°. 32′. *North, on* May 20, *two Hours before Noon.*



With the Chord of ſixty Degrees deſcribe the Circle ZENW for the Primitive; draw the Diameters ZN, EW perpendicular to each other, the firſt to repreſent the Meridian, and the other the Hori-

Horizon, From the Center H, repreſenting the South Point of the Horizon, ſet off HP equal to the Latitude, HQ equal to the Complement thereof; Hr and Hm, reſpectively, equal to the Sum and Difference of the Sun's greateſt Declination, and of the Complement of the Latitude; ~~WA, WF, EB and EG each equal to the Sun's greateſt Declination; then will the Circle deſcribed thro' E, Q and W repreſent the Equator, That thro' A, r and B the Tropic of Cancer, and That thro' F, m and G the Tropic of Capricorn,~~ alſo P will repreſent the South Pole. Moreover, thro' P, by Caſe 3. Prob. 3, let *s*P*n*, the Repreſentation of a Great Circle be deſcribed, making an Angle of thirty Degrees with the Meridian and cutting the Equator in R; thro' which, from P, draw the Right Line PRT cutting the Primitive in T; then ſet off TV equal to the Sun's Right Aſcenſion and draw the Right Line PV cutting the Equator in ♈. Thro' ♈ deſcribe (C♈D) the Repreſentation of a Great Circle to make an Angle of 23°. 29′. with EQW, then will C♈D repreſent the Ecliptic and that Point wherein it interſects the Hour Circle will repreſent the Place of the Sun at the given Time: Alſo, if thro' Z, ⊙ and N a Circle be deſcribed, it will repreſent the Azimuth Circle in which the Sun is, and I⊙ will repreſent his Altitude at the ſaid Time.

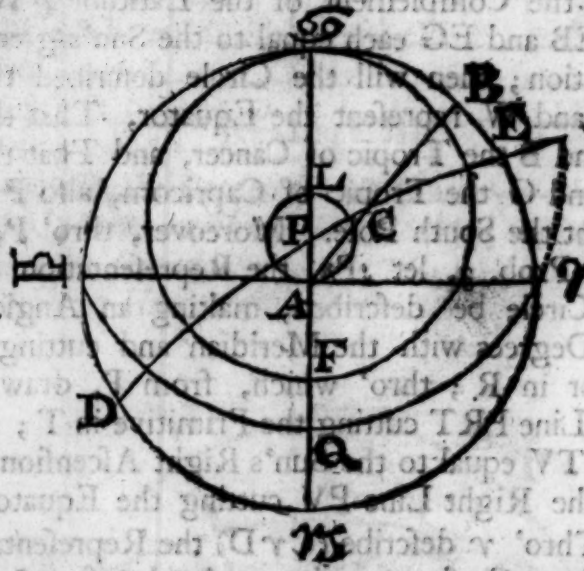
And the other way, the Difference as in Prob 2

Our Author in Case 3 of Prob. 1 and

there two are erroneous.

Example IV.

To describe the Representation of the Circles, &c. of the Sphere on the Plane of the Ecliptic.



With the Chord of sixty Degrees describe the Circle ♈ ♋ ♎ ♑ for the Primitive; draw the Diameters ♑ A ♋, ♎ A ♈ perpendicular to each other, the first to represent the Solstitial Colure, and the other the Equinoctial Colure. From the Center A, representing the North Pole of the Ecliptic, set off from the Line of Semi-Tangents AP equal to the Obliquity of the Ecliptic and AQ equal to the Complement thereof; then will P represent the Pole and the Circle described thro' ♈, Q and ♎ will represent the Equator. Make AF equal to the Difference between the Obliquity of the Ecliptic and its Complement, and, upon F 69, describe a Circle which will represent the Tropic of Cancer. Also set off AL equal to double the

the ſaid Obliquity, then a Circle deſcribed thereon will repreſent the Arctic Circle.

The chief Uſe of this Example is to project the Place of any Star: Thus, *having the Latitude and Longitude of a Star given*, ſet off, in the Primitive, ♈B equal to the Longitude from the next Equinoctial Point, according to the Order of the Signs; join A, B in which take AC equal to the Complement of Latitude, thro' P, C let DPCE, the Repreſentation of a Great Circle, be deſcribed by Prob. 6; then will C repreſent the Place of the Star, BC its Latitude, GC its Declination, PC the Complement thereof, and ♈G its Right Aſcenſion.

the said Obliquity, then a Circle described thereon will represent the Arctic Circle.

The chief Use of this Example is to project the Place of any Star: Thus, having the Latitude and Longitude of a Star given, let OP, in the Primitive, = PB equal to the Longitude from the next Equinoctial Point, according to the Order of the Signs; join A B in which take AC equal to the Complement of Latitude, thro' P, C let DPCE, the Representation of a Great Circle, be described by Prob. 6; then will the Projection the Place of the Star, BC its Latitude, and its Declination, FC the Complement thereof. And if FG its Right Ascension.

BRITISH MUSEUM 30 60 JUN

A COLLECTION *of* PROBLEMS *to be answered in the* NEXT NUMBER.

PROBLEM I. By Mr. *Thomas Moss*.

To draw a Right-Line thro' a given Point, so that the Sum of the Perpendiculars falling thereon, from two other given Points, shall be equal to a given Right-Line.

PROBLEM II. By *M.*

The Sun's Altitude being 20 Degrees; to determine the Position of a Rod of a given Length, with Respect to the Horizon (on which it stands) so that the Length of its Shadow may be a *Maximum.*

PROBLEM III. By *T.*

To determine the Time of the Day, on *February* 8, in the Latitude of 51°. 32′. North, when the Hour Angle from Noon and the Sun's Azimuth are equal to each other.

PROBLEM IV. By *S.*

The Place of the Node, and the Inclination of the Lunar Orbit to the Ecliptic, being supposed given; to find the Place of the Moon when her Declination is the greatest possible.

PROBLEM V. By *T——f——t.*

To divide a Right-Angle into four Parts, in such a manner, that the First and Second may be

equal

equal to each other, that the Fourth may be equal to the Sum of the Second and Third, and the Co-tangent of the Fourth to the Tangent of the Third, in a given Ratio; ſuppoſe that of 1 to *m*.

PROBLEM VI. By *A.*

The Lengths of three Staves being given (equal to *forty*, fifty, and ſixty Yards) it is required to place them perpendicular to the Plane of the Horizon, and at equal Diſtances from each other, ſo that Lines drawn from their Vertices, to a Point twenty Yards diſtant from the Foot of the longeſt Stave, may be equal to each other.

PROBLEM VII. By *H.*

To deſcribe the Repreſentation of a Great Circle (in the Orthographic Projection) thro' two Points, whoſe Poſitions are given with Reſpect to the Center of the Primitive.

PROBLEM VIII. By Mr. *Ra Graham*, of *Greenwich.*

To find two Numbers, in the Ratio of 5 to 7, which being reſpectively divided by 9 and 13 ſhall leave 3 and 8 for Remainders.

PROBLEM IX. By *M.*

To determine the Area of the Curve, whoſe Equation is $y = \frac{ab}{c}$ into $\left(1 - \frac{b-x}{b}\right)^{\frac{3}{2}}$.

PROBLEM X. By Mr. *Graham.*

To find the Inclination of a Plane, down which a Body, deſcending by its Gravity, ſhall ſtrike an Obſtacle, at Right-Angles to the Horizon, with the greateſt Force.

PROB-

PROBLEM XI. By *E.*

The Height of a Building, and the Length of a Spout to convey Water from the Top of it being given; to find the Angle which the Spout will make with the Plane of the Wall, so that the Water may fall at the greatest Distance possible from the Foundation of the Wall.

PROBLEM XII. By *U.*

To determine the Position of the Sail and the Course of a Ship, with Respect to the Direction of the Wind, so that she may gain to Windward the most possible in a given Time.

PROBLEM XIII. By *A.*

The Latitude of the Place being 52°. and the Sun's Declination 20°; to find the Beginning and End of the Hour, during which the Shadow of an Object will be increased to twice its Length at the Beginning.

PROBLEM XIV. By *T.*

Supposing the Radius, and the Tangent of an Arc to be given, and that two Bodies or Points *A* and *B* move at the same Time from the Extremities of the said Tangent with equal Celerities; the former along the Circumference, and the other towards the Point of Contact; to determine how far each has moved when they are the nearest possible to each other.

PROBLEM XV. By *C.*

Supposing an Annuity payable one Pound the first Year, two Pounds the second Year, three Pounds the third Year, and so on, during the Life

of

of a Perſon whoſe preſent Age is 24 Years; it is required to determine the preſent Value of that Annuity, at the Rate of 4 *per Cent.*

Problem XVI. By *Y*.

To find the Content of the ſecond Segment of a Sphere and Spheroid, formed by the Interſection of two Planes, parallel and perpendicular to the Axis of the propoſed Solid.

The END *of the* First Number.

MATHEMATICAL EXERCISES:

CONTAINING,

The Inveſtigation of the Sums of Series's.

. The Principles of Dialling.

I. Solutions to the Problems in the Firſt Number.

. A Collection of new Problems, Paradoxes, and Enigmas.

Some Obſervations on the *Ladies Diary*; together with certain Advertiſements relating thereto, printed in the *Daily Gazetteer* in *December* laſt.

By *JOHN TURNER*.

N°. II.

LONDON:

ted for JAMES MORGAN, at the *Three-Cranes* Thames-ſtreet, 1751, where may be had N°. I.

(Price ONE SHILLING.)

ERRATA, in N°. I.

IN Line 21 of the Preface, for *same* read *some*; in Line 17, p. 6, after *of* read *from the Line of Sines*; in the Note, p. 11, for *called* read *call*; in l. 28, p. 17, after *give* read *the Complement of*; in Prob. VI. for *sixty, fifty, and sixty* read *forty, fifty, and sixty*; in Prob. 9 for $1 - \frac{b}{b}\left|\frac{2}{3}\right|\frac{3}{2}$ read $1 - \frac{b-x}{b}\left|\frac{2}{3}\right|\frac{3}{2}$.

ADVERTISEMENT.

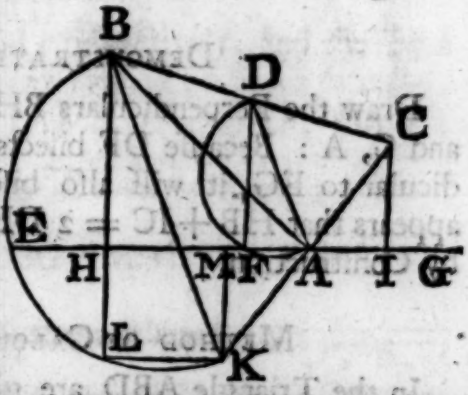
ALL Persons, inclining to encourage this Work, are desired to send their Performances (whether new Problems, Paradoxes, Solutions, &c.) Post paid, to be left with Mr. *James Morgan*, at the *Three Cranes*, in *Thames-street*, before the 10th of *August*, 1751:

The third Number being to be published on the 29th of *September* ensuing.

ANSWERS *to the* PROBLEMS, *proposed in the* FIRST NUMBER.

PROBLEM I. *Answered by the Proposer Mr.* Moss.

CONSTRUCTION.



LET A, B and C be the three given Points; join A, C in which, produced, take AK equal to AC; join B, K and thereon describe the Semicircle BEK, in which inscribe BL equal to the given Sum of the Perpendiculars; then, having joined K, L, thro' A draw the Right-Line GE parallel thereto, and the Thing is done.

DEMONSTRATION.

Upon EG let fall the Perpendicular CI, and draw KM parallel to BL (CI); then, it is plain, the two right-angled Triangles AIC and AKM are similar and equal; therefore because KM is parallel to LH, KM (CI) will be equal to LH: Moreover, since KLB is a Right-Angle and EG is parallel to LK, by Construction, BH is perpendicular to EG, and consequently BH + CI (HL) = BL, the given Sum by Construction. *Q. E. D.*

The same answered by ΦΙΛΟΣ.

CONSTRUCTION.

Let A, B and C be the three given Points; thro' the first of which the required Line is to be drawn: Join B, C which bisect with the Line AD; on AD describe a Semicircle, in which let the Line DF, equal to Half the given Line, be inscribed; then thro' A and F let the Line EG be drawn, and the Thing is done.

DEMONSTRATION.

Draw the Perpendiculars BH, CI and join B, A and C, A: Because DF bisects BC and is perpendicular to EG, it will also bisect HI; whence it appears that HB + IC = 2 DF = the given Line, by Construction. *Q. E. D.*

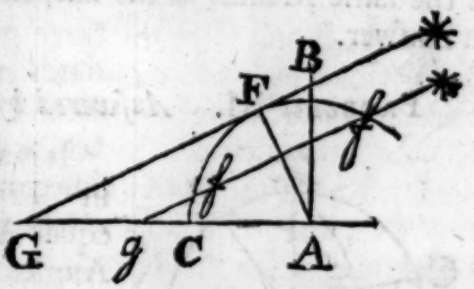
METHOD OF CALCULATION.

In the Triangle ABD are given the two Sides AB, BD and their included Angle, whence the Angles BAD, BDA and the Side AD will be found: Also, in the right-angled Triangle AFD will be given AD and DF, whence the Side AF and the Angles DAF, FDA may be had. Moreover, in the right-angled Triangle HAB, will be given the Side AB and all the Angles, whence HB and HA and consequently AI will be found.

This Problem was, also, answered by Mr. *E. R—son*, and Mr. *Harland Widd.*

PRO-

PROBLEM II. *Anſwered by Mr.* Harland Widd, *of* Whitby.



Suppoſe GB to be the Horizon and AB the Rod; make AF perpendicular to GB, and put AB $=b$, the Sine and Coſine of the Angle G ($=20°$ the Sun's Altitude) $=s$ and c reſpectively, and the Sine of BAF $=x$; Then, *by Trig.* $1 : b :: x : bx =$ BF, and, *by* 47. *E.* 1. $\sqrt{b^2 - b^2x^2} =$ AF, alſo $s : \sqrt{b^2 - b^2x^2} :: c : \frac{c}{s}\sqrt{b^2 - b^2x^2} =$ GF; whence $bx + \frac{c}{s}\sqrt{b^2 - b^2x^2} =$ BG a Maximum, therefore its Fluxion $b\dot{x} - \frac{cbx\dot{x}}{s\sqrt{1-x^2}} = 0$; whence $x = \frac{s}{\sqrt{c^2 + s^2}}$ $=$ (becauſe $c^2 + s^2 = 1$) $s = 20°$, whoſe Complement ($70°$) is the Angle B, being the Poſition of the Rod, with Reſpect to the Horizon, as required.

The ſame anſwered by Mr. E. R—ſon.

Let AB be perpendicular to the Horizon AG, and make the Angle BAF equal to the Sun's Altitude; then GAF will be the required Inclination of the Rod. For, if AF be equal to the Length of the given Rod, and FG be drawn to repreſent the Direction of the Ray interſected at F, then the Angle FAB being equal to AGF by Conſtruction, it follows that the Ray * FG will touch the Circle deſcribed, from the Center A, thro' F; which Ray, it is evident, cuts the Horizon at a greater

Diſtance, from the Point A, than any other Ray *fg* cutting the ſaid Circle.

Mr. *Moſs*'s Anſwer to this Problem was much in the ſame Manner as the laſt, and Mr. *J. H—n* ſent an Anſwer.

PROBLEM III. *Anſwered by Mr.* Widd.

It is a known Theorem, in Trigonometry, that equal Arcs ſubtend equal Angles: Therefore, ſince the external Angle Z in the Triangle BZP is, by the Problem, equal to the Angle P, it follows that ZB=SB, and conſequently that the Sun's Declination is equal to his Altitude: But the Sun's Declination on the propoſed Day is 11° 21′; therefore in the oblique angled Spherical Triangle ZBP are given PZ=38° 28′, PB=11° 21′+90° and ZB=78° 39′, whence the Angle P (=BZC) is found=~~65°~~ 54° 53′ ~~64′~~ 20′ 18″, and conſequently the Time either ~~7ᴴ 38ᴹ 16ˢ~~ in the Morning, or ~~4ᴴ 21ᴹ 44ˢ~~ 3ᴴ 39ᴹ 32ˢ in the Afternoon.

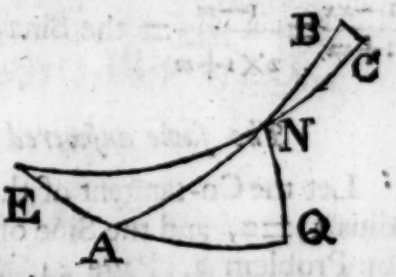
The ſame anſwered by ΦΙΛΟΣ.

Let HZON be the Meridian of the Place, PBS the required Hour Circle and ZBN the required Azimuth Circle; and from their Interſection B let the Perpendicular BC fall on the Meridian: Then all the Angles CZB, CPB, CSB, &c. being equal, by Hypotheſis, the Triangles ZBC and SBC are equal in all Reſpects; therefore we ſhall have, in the right-angled Spherical Triangle BSC, the Baſe SC (=½ ZS=½ SH+45°=~~19° 14′~~ + 45°) and SB (=78° 43′) whence the Angle CSB will be found equal to ~~65° 35′~~ 55° 7′; which converted into Time

Time gives ~~4^{H} 22^{M} 2^{S}~~ for the required Time from 12.

This Problem was anſwered by Mr. *R—ſon.*

PROBLEM IV. *Anſwered by* ΣΟΦΟΣ.



Let EQ be an Arch of the Equator, EC an Arch of the Ecliptic, and ANB an Arch of the Moon's Orbit interſecting the Ecliptic in the given Point N; alſo let NQ and BC be Arches of Great Circles, perpendicular to the Equator and Ecliptic reſpectively: Then, in the right-angled Spherical Triangle ENQ, are given the Angle E and the Hypothenuſe EN (=the Diſtance of the Node from the Equinoctial Point) whence the Angle ENQ will be found, and conſequently ANQ (=ENQ—ENA); therefore the Coſine of ANQ is to the Coſine of ENQ as the Tangent of NE to the Tangent of NA the Complement of NB the Diſtance of the Moon in her own Orbit from the Node at the Time required. Moreover, BN being thus found, in the Triangle BNC will be given BN and the Angle BNC (=the Orbits Inclination to the Ecliptic) whence NC will be had, which added to NE gives the Moon's Place (in the Ecliptic) at the Time required.

Mr. *R—ſon* anſwered this Problem nearly the ſame way as the above, and Mr. *Widd* alſo anſwered it.

PROBLEM V. *Anſwered by Mr.* T—L.

Aſſume x=the Tangent of the fourth Part; then as $1 : m :: \frac{1}{x} : \frac{m}{x}$ the Tangent of the third Part, and

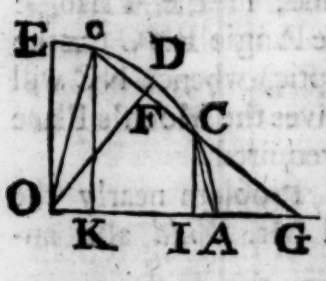
by

by Trigonometry $\frac{x-\frac{m}{x}}{1+m}=\frac{1-x^2}{2x}$ the Tangent of the firſt Part; whence $x^2=\frac{1+3m}{3+m}$, and (by Trig.) $\frac{1-xx}{1+xx}=\frac{1-m}{2\times 1+m}$ = the Sine of the firſt Arc.

The ſame anſwered by Mr. R—ſon.

Let the Co-tangent of the fourth Part (to the Radius 1)$=z$, and the Sine of the firſt Part$=x$; then, by Problem 3, Page 54. *Simpſon*'s Trigonometry and by the Nature of the Problem, we ſhall get $\frac{z-mz}{1+mz^2}=\frac{2x\sqrt{1-x^2}}{1-2x^2}$ and $\frac{x+mz\sqrt{1-x^2}}{\sqrt{1-x^2}-mzx}=\frac{1}{z}$, from which Equations we ſhall find $z=\frac{1-m-4x^2}{4mx\sqrt{1-x^2}}$, which ſubſtituted for z in the laſt Equation gives $\frac{1-m}{3x+mx}=\frac{4mx}{1-m-4x^2}$; whence $x=\frac{1-m}{2\times 1+m}$.

The ſame anſwered by ΦΙΛΟΞΕΝΟΣ.

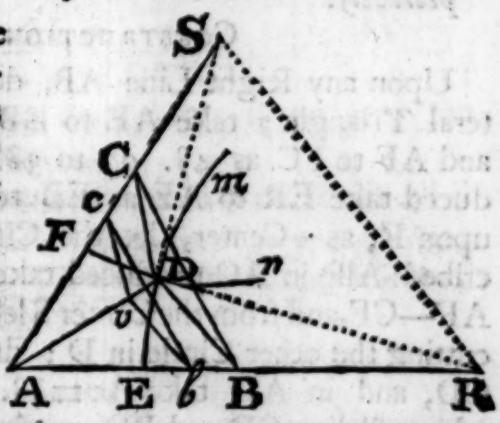


Let the Angle AOE be a Right-angle, and let the Arch AE be deſcribed; alſo let AC be the Sum of the firſt and ſecond Arcs, CD (cD) the third, and ED the fourth: Join O, D; O, c and A, C and draw cK and CI perpendicular to OA; alſo thro' c and C draw the Right Line cG, cutting OD in F and meeting OA (produced) in G. It is evident that the Arc E$c=\frac{1}{2}$ AC and conſequently that the Angle EOc (OcK) = Angle ACI; ; it will alſo appear that the Lines FG, Fc are in Proportion to the Tangents of the Arcs AD, Dc, that is as

1 to m; therefore $1+m : 1-m :: (Gc : GC : :)$ $cK : CI : : Oc\ (1) : AC = \frac{1-m}{1+m}$, conſequently $\frac{1}{2}$ AC $= \frac{1-m}{2\times 1+m}$ the Sine of the firſt or ſecond Arc.

This was anſwered, likewiſe, by Mr. *Widd.*

PROBLEM VI. *Anſwered by Mr.* R——ſon.



Suppoſe the three given Staves are erected at the Points A, B, R and that D is the other Point required; then, becauſe DA is = 20, we ſhall, *by* 47, *E.* 1, have $\sqrt{\overline{60}|^2 - \overline{20}|^2}$ equal to the Length of the common Hypothenuſe Line that is to reach from D to the Top of each Staff, which put $= b$ and we ſhall alſo have $DB = \sqrt{b^2 - \overline{50}|^2}$ and $DR = \sqrt{b^2 - \overline{40}|^2}$; therefore in the Triangle ADR there are now given AD, BD and RD, and AB = BR by the Problem, which reduces it to Problem 29, Page 175, *Simpſon's* Geometry where its Geometric Conſtruction may be ſeen.

The ſame anſwered by Sopor.

Since the Diſtance of the Foot of the longeſt Staff from the required Point is given, the Diſtance of that Point from the Vertex of the ſaid Staff is eaſily found, and will be = 63. 245; therefore the Diſtance of the Point, from the Feet of the other Staves, will be had equal to 38. 73 and 48. 99 reſpectively; whence we have another Problem being the ſame as that in Page 355 of *Simpſon's* Algebra,

bra, which I shall here put down, with his Construction.

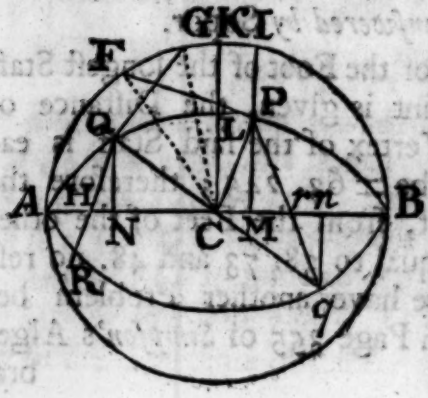
To draw three Right Lines, from the Angles of an Equilateral Triangle to a certain Point, so that their Lengths may be equal to three other given Lines a (20), b (38. 73), *and* c (48. 99) *respectively.*

CONSTRUCTION.

Upon any Right Line AB, describe an Equilateral Triangle; take AE to EB as 48. 99 to 20, and AF to FC as 48. 99 to 38. 73; in AB produced take ER to AE as EB to AE — EB, and upon R, as a Center, let the Circle EDm be described: Also, in AC produced take FS to AF as CF to AF — CF, and from the Center S let FDn be described, cutting the other Circle in D; draw AD, BD and CD, and in AD take Av = 48. 99, draw *vc* and *vb* parallel to CD and BD, cutting AS and AR in *c* and *b*; join *b*, *c* then will A*b*, *bc* and A*c* be the Distances required; each of which will be found = 58. 25: The Demonstration whereof may be seen in the above-mentioned Book.

This Problem was also answered by Mr. *Widd* and Mr. *T*— *L*—, each of whom find the Distance = 58. 479.

PROBLEM VII. *Answered by Mr.* Widd.



Since the Representations of all Oblique Circles, in the Orthographick Projection, are Ellipses (see N°. 1.) this is the same as drawing an Ellipse (the Length of whose Transverse Diameter is given) through two Points

Points whoſe Diſtance and Poſition, with Regard to the Center of the Primitive, are given: Let, therefore the Semitranſverſe CB be put $=t$, $Cq=b$, and $CP=c$, alſo $PM=y$, and $qn=x$; then, by 47. E. 1, or by Theo. 7, Book 2, *Simſon*'s Geometry, $\sqrt{bb-xx}=Cn$, and $\sqrt{cc-yy}=CM$. But, by the Property of the Ellipſe $An\times Bn : qn^2 :: AM\times BM : PM^2$, *viz.* $tt-bb+xx : xx :: tt-cc+yy : yy$; whence $y=x\sqrt{\frac{t^2-c^2}{t^2-b^2}}$; therefore $\sqrt{t^2-b^2} : \sqrt{t^2-c^2} :: x : y$, and from hence is deduced the following Conſtruction.

Join P, q and take Pr to Pq as $\sqrt{t^2-b^2}+\sqrt{t^2-c^2}$ to $\sqrt{t^2-c^2}$; then thro' r and C draw the Diameter AB and it will be the Tranſverſe Axis of the required Ellipſe.

The ſame anſwered by ΦΙΛΟΞΕΝΟΣ.

CONSTRUCTION.

From the given Points P and Q, to the Center C of the Primitive, draw PC and QC, perpendicular to which draw PF and QG, meeting the Primitive in F and G; draw QR, parallel to PC, and take QH to PC as QG to PF; thro' C and H draw ACB, then an Ellipſis deſcribed (*per* Conics) upon the Tranſverſe Axis AB, thro' the given Point P, will be the Repreſentation required.

DEMONSTRATION.

Draw QN, CK and IPM perpendicular to AB: Becauſe of the Parallel Lines PC, QH and PM, QN, it will be PC : QH :: PM : QN; but PC : QH :: PF : QG, therefore by Equality PF : QG :: PM : QN; conſequently $PF^2 : QG^2 :: PM^2 : QN^2$; Whence by Alternation and Compoſition

ſition, $PF^2+PM^2 : PM^2 :: QG^2+QN^2 : QN^2$: But $PF^2 + PM^2 = CF^2 - CP^2 + PM^2 = CB^2 - CM^2 = AM \times BM$; and in the ſame Manner $QG^2 + QN^2$ is $= AN \times BN$. Hence we have $MA \times BM : AN \times BN :: PM^2 : QN^2$, from which it is evident that an Ellipſis (the Repreſentation of an Oblique Circle) deſcribed upon AB, thro' P, will alſo paſs thro the Point Q. *Q. E. D.*

Method of Calculation.

In the Triangle CHQ, two Sides and the included Angle CQH being given, the reſt will be known; then in the Triangle PCM, the Side PC and Angles (now found) will be given, whence CM, PM and MI will be all had; whence it will be as MI : MP :: CB (CK) : CL, the Semiconjugate Axis of the Ellipſis required.

PROBLEM VIII. *Anſwered by Mr.* Widd.

Suppoſe x to be the firſt Quotient and y the ſecond, then $9x+3$ will be the firſt Number, and $13y+8$ the ſecond; but $5 : 7 :: 9x+3 : 13y+8$, and $65y + 40 = 63x + 21$, whence $y = \frac{63x-19}{65}$. Now let x be taken $= 3$, then $\frac{63x-19}{65}$ leaves 40 for a Remainder; alſo let it be taken $= 7$, and $\frac{63x-19}{65}$ will leave 32 for a Remainder; then ſay as $40-32 : 7-3 :: 32 : 16$, which added to 7 gives $x = 23$; whence $y = 22$, conſequently the Numbers ſought are 210 and 294.

The ſame anſwered by Mr. R——ſon.

Put $5x$ equal to one of the Numbers, and then $7x$ will be the other; but by the Nature of the Pro-

Problem $\frac{5x-3}{9}$ and $\frac{7x-8}{13}$ are to be whole Numbers, therefore *by Lemma Page* 166 Simpſon's *Algebra* the leaſt Value of x that will bring both theſe Expreſſions out whole Numbers will be found to be 42; whence the Numbers are 210 and 294.

The ſame anſwered by the Propoſer, Mr. Graham.

Let $9x+3$ be the leſſer of the required Numbers; then $\frac{63x+21}{5}$ will be the greater, and therefore $\frac{\frac{63x+21}{5}-8}{13} = \frac{63x-19}{65}$ muſt be a whole Number from the Conditions of the Problem: But, $\frac{65x}{65}$ being a whole Number, its Exceſs $\frac{2x+19}{65}$ above $\frac{63x-19}{65}$ muſt alſo be a whole Number; which put $= n$, then $x = \frac{65n-19}{2}$; from whence it appears that n may be any odd Number; let it be $= 1$, and then $x = \frac{95-19}{2} = 23$. Hence the two Numbers are 210 and 294.

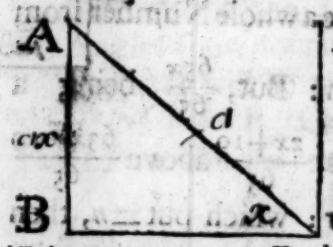
This Problem was alſo anſwered by Mr. *T. L.*

PROBLEM IX. *Anſwered by* ΦΙΛΟΣ.

Let $\left|\frac{b-x}{b}\right|^{\frac{2}{3}}$ be put $= z$, then $\left|\frac{b-x}{b}\right|^{2} = z^3$, $\frac{b-x}{b} = z^{\frac{3}{2}}$ and $b-b\,z^{\frac{3}{2}} = x$, whence $\dot{x} = -\frac{3}{2}\,b\,z^{\frac{1}{2}}\dot{z}$, conſequently the Fluxion of the Area is $= |1-z|^{\frac{3}{2}} \times -\frac{3ab^2z^{\frac{3}{2}}\dot{z}}{2c}$. Now if the Fluent of $|1-z|^{\frac{3}{2}} \times z^{\frac{1}{2}}\dot{z}$, which is the Area of the Segment of a Circle whoſe Diameter is 1 and Verſed Sine z, be denoted by A, That of $|1-z|^{\frac{3}{2}} z^{\frac{1}{2}}\dot{z}$ will, (by Compariſon with

Art. 287 *Simpſon's* Fluxions) be found $= \frac{z^{\frac{3}{2}}}{\frac{3}{2}\sqrt{1-z}} + \frac{A}{2}$; which multiplied by $-\frac{3ab^2}{2c}$ gives $-\frac{ab^2}{2c} \times \sqrt[3]{z-z^2} + \frac{3A}{2}$ for the Fluent of the propoſed Expreſſion; and this corrected, by adding $\frac{3ab^2}{4c}$ multiplied into the Area of the Semicircle, whoſe Diameter is Unity, gives the true Area required.

PROBLEM X. *Anſwered by Mr.* Moſs.



Let BC be the Horizon, AC the propoſed Plane, and DC the Obſtacle; make AB perpendicular to BC, and put AC $= a$ and the Sine of ACB $= x$; then by Trigonometry 1 (Rad.) : a : : x : ax = AB: Now when the Velocity of the Ball in the Direction BC is the greateſt it can acquire by its Deſcent along the inclined Plane AC, it will conſequently then ſtrike an Obſtacle (CD) at Right-Angles to the Horizon with the greateſt Force poſſible; but the Velocity of the Ball along the Plane CA is always as the Square Root of its perpendicular Deſcent, therefore the Velocity at C will be expreſſed by $\sqrt{BA}$ ($\sqrt{ax}$); whence 1 : $\sqrt{ax}$: : $\sqrt{1-x^2}$: $\sqrt{ax-ax^3}$ the Velocity at C in the Direction BC, which being a Maximum, its Fluxion or $(\dot{a}x - 3ax^2\dot{x})$ that of $ax - ax^3$ muſt be $= 0$; therefore $x = \sqrt{\frac{1}{3}} = ,5773$ the Sine anſwering to 35° 16′ the required Angle BCA.

The ſame anſwered by the Propoſer, Mr. Graham.

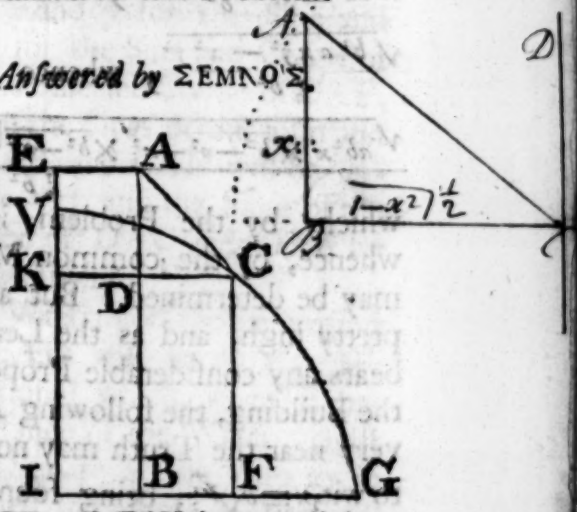
Let AC be the propoſed Plane, DC the Obſtacle and CB perpendicular to the vertical Line AB: Put

A

To find the inclination of a plane down which a Body A descending by its Gravity, shall strike an Obstacle at right angles to the Horizon with the greatest Force.

AC = 1, and AB = x, then BC = $\sqrt{1-x^2}$, and, the absolute Velocity at C being as $\sqrt{x}$, we have $1 : \sqrt{1-x^2} :: \sqrt{x} : \sqrt{x-x^3}$ the Velocity in a Direction perpendicular to the Obstacle; which being a Maximum, $x-x^3$ must also be a Maximum, and therefore $\dot{x} - 3x^2\dot{x} = 0$, whence $x = \sqrt{\frac{1}{3}}$ = Sine of 35° 16′ the Inclination required.

Mr. *R——son* also answered this Problem.



PROBLEM XI. *Answered by* ΣEMNOΣ.

Let BA be the Perpendicular Height of the Building and AC the Length and Position of the Spout, also let CG be a Portion of a Parabola described by the Water in its Fall, and whose Vertex is the Point V; let CDK be perpendicular to AB, and let CF and EUI be parallel to AB, in the last of which take VE equal to one Fourth of the Parameter and let E, A be joined. Put AB = a, AC = b, AD = x, and DC = y: By the Laws of Descent of heavy Bodies, the Velocity with which the Water leaves the Mouth of the Spout at C is the same as might be acquired by a perpendicular Descent thro AD; it is also proved that a Projectile, moving in a Parabola VCG, has a Velocity at C equal to that which might be acquired by a perpendicular Descent thro' EK; whence it follows that EK and AD are equal, and consequently that EA is parallel to the Plane of the Horizon IG.

Now the absolute Velocity at C being expressed by $\sqrt{AD}$, and that in the Direction CF by $\sqrt{UK}$, and this Proportion being also expressed by that of

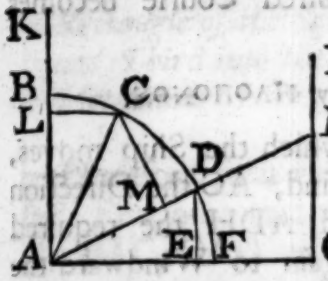
AC to AD, we have (by the Resolution of Forces) $x : UK :: b^2 : x^2$ and consequently $UK = \frac{x^3}{b^2}$ and $UE = \frac{xy^2}{b^2}$: Moreover, by the Property of the Parabola KC is $= \sqrt{UK \times 4UE} = \frac{2yx^2}{b^2}$, therefore $DK = \frac{2yx^2 - b^2y}{b^2}$; also $IG = \sqrt{UI \times 4UE} = \frac{\sqrt{b^2ax y^2 - x^2y^4}}{\frac{1}{2}b^2}$, whence BG $(= IG - IB) =$

$$\frac{\sqrt{ab^2x \times \overline{b^2 - x^2} - x^2 \times \overline{b^2 - x^2}}^{\frac{1}{2}} + \frac{1}{2}\overline{b^2 - x^2} \times \sqrt{b^2 - x^2}}{\frac{1}{2}b^2}$$

which, by the Problem, is a Maximum; from whence, by the common Method, the Value of x may be determined. But as the Equation will run pretty high, and as the Length of a Spout seldom bears any considerable Proportion to the Height of the Building, the following Approximation which is very near the Truth may not be deemed improper; to wit, $x = b\sqrt{\frac{1}{3}}$, being found by neglecting all the Terms wherein a, the Height of the Building, is not found; whence the aforesaid Expression is reduced to $\frac{\sqrt{ab^2x \times \overline{b^2 - x^2}}}{\frac{1}{2}b^2}$ whose Fluxion, *&c.* gives the Value of x as above, being the very same as that answering the Conditions of the preceding Problem.

Pro-

PROBLEM XII. *Anſwered by Mr.* R——ſon.



In the Quadrant ABF, let BA repreſent the Direction of the Wind, AC the Poſition of the Sail and AD the required Way of the Ship; draw DE perpendicular to AF and put x and y for the Sine and Coſine of BAC, z and v for the Sine and Coſine of CAD, alſo m and n for the Sine and Coſine of DAF, to the Radius 1; and then the Sines of the Sum and of the Difference of CAB and CAD will be expreſſed by $xv+zy$ and $xv-zy$, and the Sines of the Sum and of the Difference of DAC and DAF by $mv+zn$ and $mv-zn$ reſpectively.

Now the Force of the Wind upon the Sail urging the Ship in the Direction AD will be as x^2z (ſee Art. 444. *Simpſon*'s Fluxions) which, when it becomes equal to the Reſiſtance of the Water againſt the Ship, will be as the Square of the Velocity in that Direction, becauſe then the Ship's Motion will be uniform; and therefore the Velocity of the Ship in the Direction DE whereby it gains upon the Wind will be as $x\,z^{\frac{1}{2}}m$, which will be a Maximum when $3 : 1 :: xv + zy : xv - zy$ and $3 : 1 :: mv + zn : mv - zn$, as appears from Art. 428 *Simpſon*'s Fluxions, whence will be found $\frac{x}{y} = 2 \times \frac{z}{v}$ and $\frac{m}{n} = 2 \times \frac{z}{v}$, that is, in this Circumſtance the Tangent of BAC will be equal to the Tangent of DAF, and each of them double to the Tangent of CAD: Put therefore the Tangent of BAC $= t$, then by Problem 3, Page 54, *Simpſon*'s Trigonometry we ſhall have $\frac{2-t^2}{3t} = t$, whence $t = \sqrt{\frac{1}{3}} =$

.7

.7071068 answering in the Tables, nearly, to 35° 16′. the Angle of Incidence of the Wind upon the Sail, from which the required Course becomes known.

The same answered by ΦΙΛΟΠΟΝΟΣ.

Let A be the Place from which the Ship moves, KA the Direction of the Wind, AC the Direction of the Plane of the Sail, and ADH the required Way of the Ship so as to gain to Windward the most possible in a given Time. Let AG be perpendicular to AK (or GI) and from the Center A, with any Radius, let the Circle BCF be described, and let CL be perpendicular to AK, CM to AH, and DE to AG: Then, if the Force of the Wind upon the Sail, in a perpendicular Direction, be denoted by *F*, the Force by which the Wind acts upon the Sail, when the Angle of Inclination is KAC, will according to Mechanics be expressed by $\frac{\overline{CL}^2 \times F}{\overline{AC}^2}$; and it will be as AC : CM : : so is the said Force to $\left(\frac{\overline{CL}^2 \times CM \times F}{\overline{AC}^3}\right)$ the Force by which the Ship is urged in the Direction AH, which will be proportional to the Square of the Velocity in that Direction; therefore the Distance AH described in a given Time by that Velocity will be expressed by $\frac{CL \times \overline{CM}^{\frac{1}{2}} \times F^{\frac{1}{2}}}{\overline{AC}^{\frac{3}{2}}}$. But, by similar Triangles AD : DE : : $\frac{CL \times \overline{CM}^{\frac{1}{2}} \times F^{\frac{1}{2}}}{\overline{AC}^{\frac{3}{2}}}$. $\frac{CL \times \overline{CM}^{\frac{1}{2}} \times DE \times F^{\frac{1}{2}}}{\overline{AC}^{\frac{3}{2}}}$ = HG the Distance which the Ship has made to Windward in the said given Time, which being a Maximum $\overline{CL}^2 \times CM \times \overline{DE}^2$ must also be a Maximum. Therefore the Problem is reduced to this;

To

To divide a Right-Angle into three Parts, ſo that the Rectangle of the Squares of the Sines of the Firſt and Third into the Sine of the Second may be a Maximum.

Therefore, it is evident, from Art. 428. *Simpſon*'s Fluxions, that the Tangents of the ſaid Angles BAC, CAD and DAF will be proportional to each other as the Indices of the correſponding Powers of the Sines, or as 2, 1 and 2 reſpectively; and conſequently LAC and FAD equal to each other. Hence if the Tangent of BC be put $= x$, the Tangent of BC + FD will be expreſſed by $\frac{2x}{1-x^2}$, and the Cotangent thereof, or the Tangent of CD, by $\frac{1-x^2}{2x}$, therefore $2 : 1 :: x : \frac{1-x^2}{2x}$; whence $x = \sqrt{\frac{1}{2}}$.

PROBLEM XIII. *Anſwered by Mr.* T— L.

Let HZON be an Orthographick Projection of the Sphere, wherein P repreſents the Pole, Z the Zenith, HO the Horizon, EQ the Equator and *a* and *c* the Place of the Sun at the Beginning and End of the required Time. Put *a* and *n* for the Sine and Coſine of ZP (38°), *c* and *m* for the Sine and Coſine of P*a* or P*c* (70°), *d* and *p* for the Sine and Coſine of *a*P*c* (15°), alſo *x* and *y* for the Sine and Coſine of ZP*a* the Hour-Angle from Noon to the Beginning

of the required Hour; then by Trigonometry $px+dy$ and $py-dx$ will be the Sine and Coſine of ZP*c*; But by Theor. 3. Page 19, of the Projection N°. 1, we have $acy+nm$ for the Sine of *aq*, and $acpy-acdx+nm$ the Sine of *cv*; therefore, as the Lengths of the Shadows of an Object are as the Cotangents of the Sun's Altitudes, we have $\frac{2\sqrt{1-acy+nm|^2}}{acy+nm} = \frac{\sqrt{1-acpy-acdx+nm|^2}}{acpy+nm-acdx}$, whence x may be found without any further Reduction.

The ſame anſwered by Lupus.

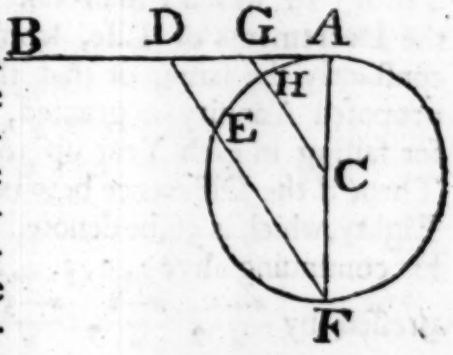
This Problem may be reſolved by ſubſtituting x for the Coſine of the Beginning of the required Hour Angle from Noon, whence the Coſine of the other Hour Angle may be found in Terms of x and given Quantities: Then, by Theor. 3, of the Orthographick Projection, the Sines of the Sun's correſponding Altitudes will alſo be exhibited, and from thence the Tangents of the ſame Altitudes; from whoſe given Ratio an Equation will be obtained, from whence x will be found. But, as the Equation thus derived is very complex, the following Method of Solution may perhaps be thought preferable.

Since the Lengths of the Shadows or the Co-Tangents of the Sun's Altitudes, at the Beginning and End of the required Hour, are in the Ratio of 1 to 2, it is evident that the Arcs themſelves muſt alſo be nearly in the Ratio of 1 to 2, and therefore the required Hour muſt end about one Hour before Sun Set, or about $6^{H}\ 51^{M}\ 4^{S}$ (becauſe, when the Sun's Declination is 20° North, the Sun in the Latitude of 52° North, ſets $7^{H}\ 51^{M}\ 4^{S}$ P. M.)

Now the Tangents of the Sun's Altitudes at the Hours $5^{H}\ 50^{M}$ and $6^{H}\ 50^{M}$ will (by Spherics) be found as .3085957 to .1458842 or as 2. 11 to 1; there-

therefore the Error is .115: But, in order to correct the foregoing Aſſumption, let the Tangents of the Altitudes at the Hours $5^H\ 40^M$ and $6^H\ 40^M$ be found, which will come out .3466068 and .1706338; and theſe are to each other in the Ratio of 2.031 to 1, therefore the Error here is .031; whence we have as .084 the Difference of the Errors is to .031 the leaſt, ſo is 10′ the Difference of the correſponding Times of the two Aſſumptions to a fourth Proportion (3′ 41″) which added to the laſt Aſſumption will give the Time ſought nearly $5^H\ 43^M\ 41$: This again corrected, by repeating the Operation, gives $5^H\ 44^M\ 59'$, or $5^h\ 45'$ for the required Anſwer, anſwering to 86° 15′.

PROBLEM XIV. *Anſwered by* ΣΕΜΝΟΣ.



Let AC be the Radius of the propoſed Circle, and AB the given Tangent; moreover let D and E be two cotemporary Poſitions of the two Bodies, whoſe Diſtance DE will be a Minimum, when the Angles D and E are equal to each other; in which Caſe, it is evident that the Line DE, produced, will paſs thro the Extremity F of the Diameter ACF. Let CG be parallel to FD and put $AC=a$, $AB=b$, and $AG=x$, then the Arch AH will be $=x-\frac{x^3}{3a^2}+\frac{x^5}{5a^4}-\frac{x^7}{7a^6}$, &c. whence AD ($=2$ AG) $=2x$, and BD ($=$ AE $=2$ AH) $=2x-\frac{2x^3}{3a^2}+\frac{2x^5}{5a^4}$, &c. alſo $4x-\frac{2x^3}{3a^2}+\frac{2x^5}{5a^4}-\frac{2x^7}{7a^6}$, &c. $=b$, or $x-\frac{x^3}{6a^2}+\frac{x^5}{10a^4}-\frac{x^7}{14a^6}$, &c. $=\frac{b}{4}$. Put

$y = \frac{a}{x}$ (= the Tangent of the Angle ACH to the Radius 1) then the Equation will become $y - \frac{y^3}{6} + \frac{y^5}{10} - \frac{y^7}{14}$, &c. $= \frac{b}{4a}$; whence by putting $\frac{b}{4a} = c$ and reverting the Series we have $y = c + \frac{c^3}{6} - \frac{c^5}{60} - \frac{2c^7}{315}$, &c. from which AE, BD and DE are alſo given.

PROBLEM XV. *Anſwered by* ΦΙΛΟΠΟΝΟΣ.

In ſolving this Problem (to avoid a laborious Calculation) I ſhall make uſe of an Hypotheſis, which, tho' not ſtrictly exact, anſwers ſufficiently near the Truth: In which I ſhall take it for granted, that the Decrements of Life, from Year to Year, are conſtantly the ſame, or that the Life on which the propoſed Annuity is granted, has an equal Chance for failing in each Year up to the Age of Eighty: Then, if the Difference betwixt his preſent Age and Eighty, which is 56, be denoted by n, the Probability of his continuing alive 1, 2, 3, 4, &c. Years, will be expreſſed by $\frac{n-1}{n}$, $\frac{n-2}{n}$, $\frac{n-3}{n}$, $\frac{n-4}{n}$, &c. reſpectively. Moreover, if r be put equal to (1.04) the Amount of one Pound in one Year, then the preſent Value of the Sums (or Rents) expected at the End of 1, 2, 3, 4, &c. Years will be $\frac{1}{r}$, $\frac{2}{r^2}$, $\frac{3}{r^3}$, $\frac{4}{r^4}$, &c. which being reſpectively multiplied by the Probability of receiving thoſe Sums, and the Products added together, there ariſes $\frac{n-1}{n} \times \frac{1}{r} + \frac{n-2}{n} \times \frac{2}{r^2} + \frac{n-3}{n} \times \frac{3}{r^3} + \frac{n-4}{n} \times \frac{4}{r^4}$, &c. = the required Value of the propoſed Annuity; which may be reduced into two Parts, to wit

$$\frac{1}{r}+\frac{2}{r^2}+\frac{3}{r^3}+\frac{4}{r^4}+\frac{5}{r^5},\ \&c.$$

$-\frac{1}{n}$ into $\frac{1}{r}+\frac{4}{r^2}+\frac{9}{r^3}+\frac{16}{r^4}+\frac{25}{r^5},\ \&c.$

Now the Sum of the firſt Part or Series, continued to n Terms (putting $\frac{1}{r}=x$) will be, by Prob. 5. Sum. Series, $\frac{\overline{x-n+1-nx}\times x^{n+1}}{\overline{1-x}^2}$ and that of the ſecond Part $=-\frac{1}{n}$ into $\frac{x+x^2}{\overline{1-x}^3}-\frac{x^{n+1}}{1-x}\times$ $n^2+\frac{2n}{1-x}+\frac{1+x}{\overline{1-x}^2}$; whence the whole Value ſought will come out equal to 196*l.* 3*s.* $0\frac{1}{2}$.

PROBLEM XVI. *Anſwered by* ΦΙΛΟΠΟΝΟΣ.

In order to give a true Solution to this Problem (without Infinite Series) it will be neceſſary, in the firſt Place, to ſhew the Manner of determining the Content of the Part of the Surface of a Sphere included between two Leſſer Circles thereof, whoſe Poles are 90 Degrees diſtant from each other. Thus let RTS and RVS be the two Parallels (or Leſſer Circles of the Sphere) whoſe Poles are Q and P, and let PR, PS, QR and QS be Arcs of Great Circles drawn to the Interſections of the ſaid Leſſer Circles: In the Triangle PQR (or QPS) are given the Side PQ $=90°$, the Side PS equal to the given Diſtance of the Parallel RVS from its Pole, alſo QS equal to the Diſtance of the Parallel RTS from its Pole; whence all the Angles P, Q and S may be found, and conſequently the Exceſs

Exceſs of all the Three above two Right Angles: Then, it will be by Art. 193. *Simpſon*'s Fluxions, as 720° is to the ſaid Exceſs, ſo is the Superficies of the whole Sphere to the Meaſure of the Superficies PSQ. Moreover, as the Diameter of the Sphere is to the Verſed Sine of the Arc PV, ſo is the Superficies of the whole Sphere to the Superficies of the Part cut off by the whole Parallel whereof RVS is a Part; and as 360° is to the Angle SPV, ſo is the Superficies laſt found to the Superficies PVS. In the ſame Manner the Superficies SQT will be found, and the Exceſs of theſe Two above PSQ will give the Content of TVS, whereof the Double is the whole Content (of STRVS) required.

Now let ABHC be any propoſed Hemiſphere, and EFG a ſecond Segment thereof by the Planes of the two Circles, whoſe Radii are GK and EI; then the Diſtance of the ſaid Circles from their reſpective Poles B and H being given, the convex Surface of the ſaid Segment EFG will be found from above; which, multiplied by $\frac{1}{3}$ of the Radius of the Sphere, gives the Content of the Part EAFGE: But the Content of the Part AFG will be found by multiplying the Segment of the Circle whoſe Radius is GK and Verſed Sine GF by $\frac{1}{3}$ of the Perpendicular Altitude AK. And in the like Manner, the Content of the Part AFE will be found: Therefore the Sum of theſe Two ſubſtracted from the Part EAFGE will give the required Content of the Segment EFG.

If

* Prob 1. Case 3 & 4

To find the sum and difference of the proposed parallels distance from the pole and pole of the Prime Vertical which taken from the Scale of Semi-Tangents, and set off in the line of meridian will give E & G the extremities and if on G as a diameter, a circle be described will be the parallel required.

NB. If the Circles distance from its pole, be less than the poles distance from that of the prime the point G will be of the same side O with E.

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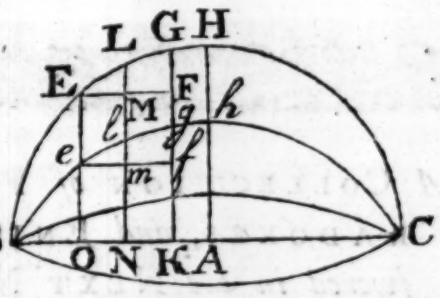

If now AB*b*C be ſuppoſed a Semi-Spheroid, whoſe Axis is BC, and ABHC be a Hemiſphere circumſcribing it, it is demonſtrable that the ſecond Segment *efg* of the Semi-Spheroid is to the correſponding Segment EFG of the ſaid circumſcribing Hemiſphere, as the whole Spheroid is to the whole Sphere, or as the Square of (2 A*b*) the leſſer Diameter of the Spheroid is to the Square of (BC) the greater Diameter.

For, if a Plane, parallel to AH, whoſe Radius is LN, be ſuppoſed to interſect both the Segments at any Diſtance from OE or KG, the Areas of the Sections LM and *lm* will evidently, from the Nature of the Figure, be to each other as the Circles themſelves, whereof they are Parts, and whoſe Semi-Diameters are LN and *ln*; and theſe Circles being every where to each other as LN^2 to lN^2, or as AH^2 to Ab^2, the Thing is manifeſt.

A COLLECTION *of* PROBLEMS, PARADOXES, *and* ENIGMAS; *to be anſwered in the* NEXT NUMBER.

PROBLEM XVII. *By Mr.* Thomas Marſhall, *of* Blenkland.

TO determine two Numbers ſuch, that being reſpectively added to, and ſubſtracted from, 48, the Sums ſhall be in the Ratio of 5 to 6, and the Differences in the Ratio of 3 to 2.

PROBLEM XVIII. *By Mr.* Moſs.

The Area of a Right-angled Triangle, and the Difference between the Hypothenuſe and the Perpendicular falling thereon, from the Right Angle, being given; to determine the Triangle.

PROBLEM XIX. *By* Nauticus.

A Ship bound to Windward, then at E.N.E. ſails with her Larboard Tacks on Board 12 Miles; ſhe then tacks, and having run 16 Miles more, it appeared, that, upon both Tacks, the Difference of Latitude made good was 6 Miles, to the North, and the whole Departure 14 Miles, to the Eaſt. What was the Courſe ſailed on each Tack; and how near did ſhe lie to the Wind?

PROBLEM XX. *By* X. Y.

Suppoſing $x^3 - y^3 = 2x^2 - 2y^2 = 3x - 3y$; it is propoſed to find from thence the Values of x and y.

Dividing the above equations by $x-y$ PRO-

given .... $x^2+xy+y^2 = 2x+2y = 3$.

PROBLEM XXI. *By Mr.* J. H——n.

Required the three leaſt Numbers, which being divided by 2, 3, 4, 5, 6, 7, 8, 9 ſhall leave the Remainders 1, 2, 1, 4, 5, 1, 5, 2 reſpectively.

PROBLEM XXII. *By* Nonſenſienſis.

From what Height muſt a Ball, of 4 Ounces, fall, to have the ſame Force, on a Plane whoſe Inclination to the Horizon is 40 Degrees, as another Ball of 9 Ounces, falling from the Height of 12 Feet, has on a Plane whoſe Inclination is 25°?

PROBLEM XXIII. *By Mr.* Graham.

The Ratio of the Sines of two Arches, as alſo the Ratio of their Coſines, being given; to find the Arches themſelves.

PROBLEM XXIV. *By* Nauticus.

A Ship, bound to a Port lying N.E. at the Diſtance of 66 Miles, ſails a certain Courſe, till, by Obſervation, her Difference of Latitude is found to be 24 Miles; ſhe then alters her Courſe, ſailing directly towards the deſigned Port; and upon her Arrival There it appears that the whole Diſtance run is 78 Miles: Now the Queſtion is to find each Courſe and Diſtance?

PROBLEM XXV. *By Mr.* William Toſt.

A Ship ſailed from the Lizard, in *England*, Latitude 50° N. on the Arch of a Great Circle S.S.W, ½ W. until ſhe met a Ship at Sea that had ſailed alſo on the Arch of a Great Circle from the Iſland of *Barbadoes*, Lat 12° 58′ N. Long. from the Lizard 53° 40′ W. directly E.S.E; it is required to find

 the

* The Quest. was designed by the Proposer to have been according to Mercator but was mistaken by the publisher.

the Latitude and Longitude of the Place, where the Ships met, from the Lizard.

Problem XXVI. *By* ΦΙΛΟΣ.

Suppoſing the Inclination of the Lunar Orbit to the Ecliptic to be 5°, and the aſcending Node in 15° of ♉; to find the Latitude of the Place where the Moon riſes (for ſeveral Nights) with the ſame Point of the Ecliptick, or about 4 Minutes ſooner each Night than the preceding Night.

Problem XXVII. *By Mr.* T—— L.

A Gentleman put 2000*l.* into the Hands of a Friend, in Order to receive a weekly Allowance; what muſt that Allowance be ſo that, at the End of 7 Years, the whole Sum with its Compound Intereſt, at the Rate of 4 *per Cent.* may be entirely exhauſted?

Problem XXVIII. *By* X. Y.

What are the ſeveral Values of x, y, z and u, in the following Equations, *viz.* $x + y + z + u = 24$, $x^2 + y^2 + z^2 + u^2 = 178$, $x^3 + y^3 + z^3 + u^3 = 1476$, and $x^4 + y^4 + z^4 + u^4 = 13042$?

Problem XXIX. *By Mr.* Moſs.

Suppoſe a Ball to deſcend in the Curve of a given Cycloid, whoſe Axis is perpendicular to the Plane of the Horizon; it is required to find in what Part of the ſaid Curve the Ball approaches the faſteſt towards the Horizon?

Problem XXX. *By* Curioſus.

It is reported by Travellers that the greateſt *Egyptian* Pyramids, at certain Seaſons of the Year, caſt

caſt no Shadow: The Truth whereof may be confirmed by Aſtronomical *Calculations*. In Order to which, let us ſuppoſe (according to the Obſervations of our Countryman *Greaves*) that the Perpendicular Height of the greateſt of the ſaid Pyramids is 481 Feet, and each Side of its Square Baſe (facing the four Cardinal Points) 693 Feet, and that it ſtands in 29° 40′ No. Lat. From hence it is propoſed to find the Day when it firſt appears without a Shadow, and alſo how long it is poſſible to continue without a Shadow, at one Time, when Days are at the longeſt?

Problem XXXI. *By* Curioſus.

Travelling in the Night between the 15th and 16th of *October* in the Year 1749, I obſerved, in the Heavens, a remarkable Phenomenon, and being very deſirous to be able to tell the Hour of the Night, I took particular Notice of two remarkable bright Stars, which I knew to be Sirius and the Planet Jupiter; which Stars, as near as I could poſſibly judge, had at that Time exactly the ſame Altitude. From whence and the Latitude of the Place, which was 52° N, the Hour of the Night may be found and is here required?

Problem XXXII. *By* W. Trott.

Suppoſe a Beam, having two unequal Weights ſuſpended to the Ends thereof at equal Diſtances from the Fulcrum, is ſuffered to deſcend (or move about its Axis) from a Poſition parallel to the Horizon, till it becomes perpendicular thereto; the Time of Deſcent is required, the Length of the Beam being 3 Feet, and the Weights 12 and 18 Pounds?

PROBLEM XXXIII. *By Mr.* P. Kay.

Of all the Cones of the ſame given Solidity (*S*) to determine that which, being ſuſpended at its Vertex, ſhall vibrate in the ſhorteſt Time.

PROBLEM XXXIV. *By* Caliban.

If one good Draught of a Bottle of Brandy ſinks it $\frac{1}{5}$ of the whole Depth*; How many ſuch Draughts will there be before the Bottle is quite empty?

* *We are to ſuppoſe that Mr.* Caliban *means, that, when the Axis of the Bottle is placed parallel to the Horizon, the Surface of the Liquor will be ſunk* $\frac{1}{5}$ *of the Bung Diameter; We ſhall alſo take it for granted that his Bottle is in the Form of the Middle Fruſtrum of a Spheroid, whoſe Head and Bung Diameters are in Porportion as* 5 *to* 6.

PROBLEM XXXV. *By* J. T. (*from* Ladies Diary *unanſwered.*)

To find the Nature of the Curve (deſcribed by a Body giving uniform and direct Chace to another) ſuppoſing the Body purſued to move uniformly in the Periphery of a given Circle.

Two PARADOXES *propoſed by Mr.* Graham.

I.

There are two Places, on the Surface of the Globe, that bear directly North from each other.

II.

There are likewiſe two Places on the Globe, the firſt of which bears directly North from the Second; notwithſtanding the ſecond bears directly Weſt from the firſt.

Two

If point, P, or 2, being given, either within or without the primitive, to find another point 2 or P, so, posited that any circle K M 2 N which passes thro' 2 and P shall be the representation of a great circle.

From the given point thro' the center draw P2 at pleasure and let

C be a radius of the primitive perpend to P2; draw a line from the given point P or 2 to C, perpend. to which at C, draw a line to meet P2 in point 2 or P not given, and it shall be the point required.

Demonstration

Let P2 at right angles with K L any where which as at Q let the center of a circle 2 N P M then which passes thro' P & 2, and let M be one of the points in which it intersects the periphery of the primitive, and thro' O draw M N a chord of the circle 2 N P M; then $OM \times ON = OP \times O2 = CQ - OM^2$; therefore $OM = ON$, and the point N is in the primitive opposite to M, whence M P N 2 is the representation of a great circle

2 D.

Corollary

Hence the representation of a great c
may be drawn to pass thro' two given
points P or 2, and A: For to one of the g
Point suppose P or 2, find 2 or P, its
reciprocal as above, and a Circle des
thro' P, 2 & A will be the representation
required. Or

draw P2 and OC, and join C to the given
Point P or 2 as above; let F L bisect this
at right angles in F and meet P2 in L
draw K L ‖ to OC; find G in K L the c
of the Circle MPAN passing thro' A &
whichever is given, and it shall be the
representation required.

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Two Enigmas propoſed by J. S——s.

I.

1. Sometimes I'm gilded o'er with Gold,
To make more ſplendid Show,
Sometimes in meaner Dreſs I'm ſold,
As the fair Ladies know.

2. Yet the ſame Purpoſes I ſerve,
However meanly dreſt;
And, in my Bowels, carry Food,
To entertain my Gueſt.

3. Sometimes I am a Priſon ſtrong;
Sometimes laid by neglected:
I'm neither Male nor Female found,
For which, ne'er leſs reſpected.

4. And Ladies if you ſhould me find,
You'll ne'er the more diſlike,
For, if I am inhabited,
Songs may your Pains requite.

II.

1. Oh how delightful is my Place,
What Virgin Charms do I embrace,
The ſtrongeſt Charms that Beauty boaſt;
By Fate ſo happy is my Lot,
But ſtill, let it not be forgot,
How I, at other Times, am croſt.

2. For when the Fair take me in Hand,
They beat, they ſhake, they ſtroke, then ſtand,
To lay me ſmooth and fit for Sight.

And

And I, this Uſage, don't regard,
Since by it I'm better prepar'd,
To pleaſe the Fair again at Night.

3. The hardier Sex make much of me,
And ſay with Them I well agree;
So that with both I'm in repute:
And Ladies I no Queſtion make,
But, in the Morning when you wake,
That you will quickly find me out.

Obſervations, by John Turner, *on certain invidious Aſperſions on Mr.* Simpſon's *Doctrine and Application of Fluxions; publiſhed in the* Monthly Review *for* December *laſt, by* Cantabrigienſis.

Theſe *injurious* Reflections, which are mentioned by the Author of the *Gentleman's Palladium (at p. 53)* as candid Obſervations and excellent Remarks are judged to be the Productions of that Author; the remarkable Confuſion of Ideas, the peremptory Manner in which they are delivered, together with a Leven of Gall which runs thro' the Whole, and correſponds ſo well with his avowed, and *inveterate* Enmity to Mr. *Simpſon*, are ſuch ſtrong Circumſtances in Favour of this Opinion, that, I dare ſay, no Body, that knows him, will aſcribe them to any other Perſon. His Modeſty, as well as his Deſign, appears in his very firſt Paragraph; wherein he ſuggeſts, that the World, from a *few* General Obſervations (which he is to give) will be enabled to form a Judgment, not only of Mr. *Simpſon*'s Fluxions, but likewiſe of the Writer's Abilities, with regard to *Method*, *Conciſeneſs*, *Perſpicuity*, *Accuracy*, and *Judgment*. It ſeems plain from hence, that our *candid* Obſerver, notwithſtanding his fair Pretences, had far other Views than giving an impartial Account of the Work under

der Conſideration —— Elſe, why are ſuch a Number of particular Qualifications muſtered together, and ſo *emphatically* enumerated, if not with a Deſign to make ſhort Work of it, by inſinuating to his Readers that Mr. *Simpſon* had no juſt Pretenſions to *Any*.

Hard, indeed, would be the Caſe of an Author, was the Judgment of Mankind to be directed by *ſuch* Commentators! But the Publick will judge for themſelves; and They are the proper Judges: The Reception which Mr. *Simpſon*'s Works, in general, have met with muſt, at leaſt, be as ſtrong an Argument of the Author's having *ſome* Merit, as this Commentator's Inſinuations can poſſibly be, of his having *none at all.* ——— But to proceed to Particulars: The firſt Objection of our Critic is *againſt the Confuſion with which, he pretends, many Things are treated.* In anſwer to which I affirm that he merits the ſevereſt Contempt for endeavouring to ſtigmatize this Author for that Confuſion which exiſts in his own Imagination, and no where elſe: For I ſhall prove to a Demonſtration, by and by, that he has been ſo far from being able to make good this Charge, that the very Inſtances he has produced in Support of it ſerve only, to ſhew his own extreme Ignorance, even, in the firſt Principles and moſt common Part of the Subject.

His Aſſertion that Mr. *Simpſon had miſtaken the Effect for the Cauſe*, is equally unjuſt and without Foundation; and what follows thereon, downright Nonſenſe. Mr. *Simpſon* builds upon his own Definition; which, he tells us, *himſelf*, is not exactly the ſame as That of Sir *Iſaac Newton*, and has alſo ſhewn wherein the Difference conſiſts: He ought not, therefore, to be accountable for an injudicious Commentator's not being able to reconcile the one with the *other*.

But the moſt injurious Reflection that the Malice and ill Nature of our Critic could poſſibly ſuggeſt is, his Imputation of Plagiariſm. He has had the Confidence to aſſert, without the leaſt Reſerve, that all Mr. *Simpſon* has given for determining the Fluxions of the Sides and Angles of Spherical Triangles, *is taken from Mr. Cotes's Eſtimatio Errorum.* Tho' this Book is in every Bodies Hands, and not one Half of the Things given by Mr. *Simpſon* are to be found in it; And tho' his Method is, alſo, quite different.

Thus much in anſwer to his General Charge, concerning *Confuſion*, *Miſtakes*, and *Plagiariſm*.—I come now to take ſome Notice of the Particulars produced by him to prove that the *Author* has neither *Method*, *Conciſeneſs*, *Perſpicuity*, *Accuracy* nor *Judgment*; where I ſhall make it appear, to a Demonſtration, that the *Commentator* has neither *Veracity*, *Preception*, *Reaſon*, *Senſe*, nor *Modeſty*.

Here his Remarks on the Author's Definition of a Fluxion firſt demand our Conſideration: Mr. *Simpſon* makes it to be, "the Magnitude by which a flowing Quantity *would be* uniformly increaſed in a given Time." This Definition the *Critic* repreſents as a *very odd one*; and with regard thereto advances the two following, *extraordinary*, Poſitions.

1. That, in Quantities uniformly generated, the *Fluxion* muſt (according to the ſaid Definition) be the *Fluent itſelf*, or elſe a Part of it.

2. And that, in other Quantities generated by a *variable* Law, the Fluxion will not be a *real*, but an *imaginary* Thing.

To the firſt of theſe Objections I anſwer, that the *Fluxion* is neither the *Fluent itſelf* nor a Part of it: It is a Quantity of the ſame Kind with the Fluent; but the Fluent being the Quantity *already* produced by the generating Point, Line or Surface, ſuppoſed

ſtill

ſtill in Motion, and the Fluxion *what* will ariſe, hereafter, from the Continuation of that Motion; the latter can no more be denominated a Part of the former than the enſuing Hour a Part of the Time paſt.

But his ſecond Obſervation is a ſtill more glaring Inſtance of his Diſingenuity, and Want of Judgment. Does it follow, becauſe a Body, really, moves over a certain Diſtance, in a given Time, with an accelerated, or a retarded Velocity, that there is no Diſtance over which it might paſs in the ſame Time, with its firſt Velocity uniformly continued.

The Space over which a Body would uniformly move with ſuch, or ſuch, a propoſed Velocity, is no leſs *real* becauſe no Part of it is *actually* deſcribed with that Velocity. — It is allowed, by All, that a Body, deſcending by its own Gravity, acquires, in the firſt Second of Time, a Velocity ſufficient to carry it, *uniformly*, over a Space of 32 Feet in the next Second: Now this Diſtance of 32 Feet, is, according to our *Sagacious* Critic, no *real Quantity*, but, *an imaginary Thing*. Strange Perverſeneſs, and Stupidity! not to diſtinguiſh the Difference between the Meaning of *a real Diſtance* and *a Diſtance really deſcribed*.

The next Inſtance he brings to prove the Author's Want of *Judgment*, *&c.* relates to the Reſolution of the fluxional Equation $c\,x^2\dot{x} + y\dot{x} = a\dot{y}$: Whoſe Fluent the Author makes to be $y = -c \times \overline{xx + 2ax + 2aa + dM^{\frac{x}{a}}}$; *where M denotes the Number whoſe Hyperbolical Logarithm is Unity, and* d *any conſtant Quantity which the Nature of the Problem may require.* With reſpect to this Solution the Author intimates that, though the Conditions of the fluxional Equation firſt propoſed are anſwered by the Equation $y = -c \times \overline{x^2 + 2ax + 2aa}$ (where the

Exponential d $M^{\frac{z}{r}}$ is wanting) yet ſuch a Solution can be of no real Uſe in the Reſolution of Problems, ſince the ſaid Equation, or Fluent (without the *Exponential*) is not capable of ſuch a Correction as the Nature of the Problem may require.

Now let us ſee what our Critic ſays to This. He affirms, that if the Fluxion of the laſt Equation, without the Exponential, produces the fluxional Equation propoſed, *then the Exponential is not neceſſary, but, that, the Reſult will be according as the Nature of the Problem directs.*

It is impoſſible for any One to be guilty of greater Inconſiſtencies than are here advanced. He allows that the Nature of the Problem may require a particular Correction (which is juſt) yet he affirms that the Solution is perfect, tho' incapable of ſuch a Correction, and in reality no Solution at all. — As in moſt Caſes x and y are equal to Nothing at the ſame Time, theſe Caſes Mr. *Simpſon* has illuſtrated by an Example. From whence any one, who has the leaſt Penetration, will eaſily ſee what Uſe to make of the *Exponential* in any other Caſe. By Means of which, *only*, (and not by a conſtant Quantity) the neceſſary Correction can be made, let that neceſſary Correction be what it will.

Our Critic, in his next Article, abates ſomewhat of his uſual Poſitiveneſs: He deſires to be informed *How* (at p. 455) *his Author comes to make* $x = \frac{f}{1+g}$, *&c.* Now had he been capable of reading but two Pages farther, he would, not only have found the Reaſon he wants, but muſt, in ſpite of his Prejudice, have approved of the Author's Invention. But he ſays, What follows *is very obſcure and intricate*; in which he is certainly right, if he *only* ſpeaks according as it appears to *him*. Nay! I will allow that the Subject *itſelf* is *intricate*; being One of

of the moſt difficult, as well as one of the moſt important Enquiries in Natural Philoſophy, and ſo much above the Commentator's Level, that an ingenious Author, to whom he, every where, ſhews a remarkable Attachment has repreſented, What Mr. *Simpſon* has (ſince) demonſtrated *thereon*, as a Thing not to be done by Any.

Our Critic farther obſerves, that the Author has, in the Courſe of his Work, *ſeveral Problems which do not at all belong to Fluxions*. This Aſſertion, in one Senſe, may be allowed to be juſt: But, then, he has not thought proper to obſerve, *alſo*, that, thoſe very Problems are given, as introductory to Others, where both *They* and *Fluxions* are *abſolutely* neceſſary. As to the, *Whole*, laſt Section, it is to be conſidered in the Light of an Appendix; as the Title thereof, *the Reſolution of Problems of various Kinds*, ſufficiently confirms.

I have now but one more Particular to take Notice of; and this relates to our Critic's *admirable* Sagacity in finding out the *true* Reaſon why Mr. *Simpſon* aſſumed the Title of, the Doctrine and Application of Fluxions to his late Work: This he ſignifies *was done to give it a Character and make it ſell, as being Mr.* Emerſon's *Title to a Treatiſe of his on the Subject*. —— As it would be extremely hard to find proper Words to expreſs the Ideas that muſt ariſe in the Mind of an intelligent Perſon, on this Information, I ſhall leave my Readers to make what Remarks they pleaſe on ſo well grounded and profitable a Diſcovery. One Thing, however, I muſt obſerve — If Mr. *Simpſon* did really act with thoſe Views, in order to impoſe upon the Publick, he is certainly very ungrateful, in forgetting, ſo ſoon, the favourable Reception his former Book on the Subject met with; tho' unaccommodated with a Title of Mr. *Emerſon*'s.

The extravagant Encomiums which our Critic, every where, beſtows on this Gentleman, at Mr. *Simpſon*'s Expence, cannot be unobſerved: And many Perſons have, to my certain Knowledge, been, from thence, induced to think (eſpecially Thoſe who know how incapable he is of carrying on the Diary and Palladium without the Aſſiſtance of ſome abler Hand) that he has an Intereſt in courting the Favour of that Gentleman, and that he is actually aſſiſted by him. — Whether this Opinion has a ſufficient Foundation, and whether a General Belief of it can be of any Service to the ſaid Gentleman's Character, I do not take upon me to determine. One Thing, however, I ſhall take the Liberty to obſerve. — As all Men are liable to Errors, it is prudent to avoid all Cauſe of Miſunderſtanding with Thoſe, who are the moſt capable of detecting our Weakneſſes.

I ſhall now oblige the Curious with certain Advertiſements, ſent by an unknown Perſon; the four Firſt whereof appear to have been printed in the *Daily Gazetteer* for *December* laſt; but in what Paper the laſt was printed, or whether, ever printed at all, I am uninformed.

JOHN TURNER.

N°. I. (*December* 4.)

To the Worſhipful Company of Stationers.

Gentlemen,

I Hope you will pardon the Liberty of this Application, which I was induced to on the following Account: The *Ladies Diary*, as well as every common Almanack, is, I am informed, a Part of your Property; which I cannot help thinking is treacherouſly undermined by the Compiler thereof, whoever he is, whom you entruſt. This Perſon, at the ſame Time, carries on another Work of the like

like Nature upon his own Foundation, under the Name of the *Palladium*, which you may probably be apprized of: But you may not perhaps be ſo well informed, that all the beſt Materials deſigned for the Diary by the Contributors, are ſelected out, and reſerved by him for his ſaid *Palladium*. This, tho' perhaps below your Notice, is a Matter of Fact well known to the Contributors, whereof I own myſelf to be one; and have more than once been diſappointed in not meeting my Mite in the Place where I expected to find it; which, to confeſs the Truth, is the principal Reaſon of my taking this Freedom. Any Perſon, by comparing together the two Performances for the enſuing Year, will ſoon be convinced, that tho' the beſt of them is far from being perfect, yet the *Diary* is by many Degrees the worſe; and I will venture to ſay, the very worſt that has yet appeared. So much has this once uſeful Performance ſunk in its Value, thro' the Inability of an unskilful or deſigning Conductor on the one Hand, and the *Palladium* drawing away its beſt Support on the other.

Honeſtus.

N°. II. (*December* 6.)

To the Worſhipful Company of Stationers.

Gentlemen,

THE Author and Compiler of the *Palladium*, and *Ladies Diary*, in Anſwer to the Advertiſement of the 4th Inſtant, ſigned *Honeſtus*, does not take upon him to determine whether the Author of that Advertiſement is a *Knave* or a *Fool*, *Spectre*, *Dæmon*, or *Apparition*; and therefore thinks himſelf not culpable for what is ſaid of him in the Dark, without Proof. If he is an honeſt Man; no Shadow, but a real Subſtance, actuated by the Principles of Morality, he is deſired to come forth into

into the Light, and make good his Aſſertions, *viva voce*, if he can, in the Preſence of you Gentlemen, and before the Face of the little Compiler, who challenges him to appear before you in the Cauſe of Truth, at *Stationers-Hall*, or at any other convenient Place he ſhall appoint, to diſcuſs the Matter of Fact before impartial Judges. And that he may have no Excuſe for Neglect, he is hereby promiſed a Reward for the Diſcovery of his Name and Perſon, ſo that he may be ſpoke withal, meaning Mr. *Honeſtus*. And any honeſt Man that will diſcover who this ingenious Writer is, ſhall receive ample Satisfaction, upon certain Proof of his Identity. For the Goodneſs of the *Ladies Diary* for 1751, which this Author of Conſequence would recommend himſelf to improve, the preſent Author of it appeals to your Judgment; he having always acted with the ſame Integrity as if the ſaid *Diary* had been his own Property; when that Honour fails, or he betrays his Truſt, he expects the Character which now belongs to this ignominious and unfair Accuſer. His Mite was not worth preſerving. It is a Rule to oblige the Public with what is preferable, rather than an inſignificant Contributor, who may enquire for the Author of the *Palladium* of Mr. *Simpſon*, at *Stationers-Hall*.

N. B. The firſt and laſt Sheet of the *Diary*, made to contradict each other about the Delivery of Letters, occaſioned by the Printer, the Author does deſire they ſhall all be left at Mr. *Price's* in the *Poultry, London*.

N°.

N°. III. (*December* 13.)

To Χρονομονοπυβλιν, *the Compiler of the* Ladies Diary.

SIR,

WHEN a Diſputant leaves the Point in Queſtion, and falls to Swaggering and Raving, it is a certain Sign that he either has a bad Cauſe to manage, or wants Senſe and Temper to make the beſt of a good one. The former of which I take to be your Caſe; ſince, notwithſtanding all your ill mannered Bluſtering, you either entirely forgot, or *wiſely* omitted, to take Notice of the principal Matter urged againſt you. — Could you have fairly made it appear that you had never converted to your own Uſe what was deſigned for the *Diary*, you need not have been under ſuch furious Agitations about a *Knave, Fool, Spectre, Dæmon*, &c.

You inſiſt much on your Honour and Integrity, which I ſhall not now diſpute; but if the World, as well as myſelf, ſhould acquit you of ſiniſter Deſigns, how then is the remarkable Badneſs of the *Diary* under your Hands to be accounted for? Either your Integrity or your Judgment muſt be called in Queſtion: Since, whatever you may think or pretend, that once uſeful Work, is now no better than an incorrect poor Performance, abounding with the moſt enormous Abſurdities. One or two of which, with your Leave, I ſhall here take the Liberty to produce *as a Specimen.*

The Firſt is your Third Mathematical Queſtion for the enſuing Year, which runs thus:

From what Height muſt a Ball of four Ounces Weight be let fall, to have $49\frac{61}{100}$ *Pounds Force on an inclining Plane, whoſe Angle of Incidence is* 40 *Degrees.*

This

This Queſtion, which is no better than downright Nonſenſe, you *(who always prefer what is beſt)* have thought proper to oblige the Publick with: Tho' every one, the leaſt acquainted with the Principles of Natural Philoſophy, knows, that the Forces of Preſſure and Percuſſion can no more be compared together, than a Line and a Surface.

Another of your Queſtions, ſigned *Upnorenſis*, is as follows:

To determine the Sides of the leaſt *right-angled Triangle in whole Numbers, whoſe Legs are in Proportion as 7 to* 11.

From this Queſtion I am afraid the World will be apt to conclude, that you are about as well verſed in *Euclid*'s Elements, as in Sir *Iſaac Newton*'s Philoſophy. What Pity it is, that neither you, nor your ingenious Correſpondent, ever heard talk of ſuch Things as incommenſurable Quantities. Had you been the leaſt inſtructed herein, you might have diſcovered that this Queſtion is alſo impoſſible.

A Multitude of other Inſtances of the like Nature might be brought againſt you, did it ſeem neceſſary: But what is already advanced may ſuffice to ſatisfy my firſt Aſſertion, *viz. That the Diary had greatly ſunk in its Value, either through the Inability, or Deſigns, of the Conductor.*

I am, Sir,

Honeſtus.

P. S. When χρονομονονπυβλιχ☉ has anſwered the above Objections, and ſome few others, to the Satisfaction of the Publick, the Spectre that has ſo much diſcompoſed him, engages to appear in his own proper Shape without the Temptation of the *promiſed Reward*, offered for ſuch Diſcovery.

N°.

N°. IV. (See *December* 14.)

To the Worſhipful Company of Stationers.

IN Anſwer to Yeſterday's Advertiſement by the pretended *Honeſtus*, it is only proper to obſerve, that as he is either *afraid*, or *aſhamed* to comply with the *Honour* requeſted (*See Gazetteer of the 6th*) by appearing in *propria Perſona*, before you and the Perſon whom he accuſed, he is unworthy farther Notice. For the Solution of the Problems which this *Sham Litigant* wants to be ſatisfied in, he is referred to the next *Ladies Diary*: It being inconſiſtent to engage in a Diſpute with an unknown Perſon, at a certain Expence, or to enter the Liſts with one who deſerves only Contempt or Pity for his groſs Ignorance and abandoned Conduct; acting like an *Aſſaſſin* in the Dark! and abjectly and cowardly refuſing to ſhew himſelf by Day-Light. As the *Ladies Diary* depends more on Amuſement for the *Fair Sex*, than on the Propriety of Mathematical Problems, were his Objections to two Problems juſtifiable, it would concern but about five Hundred Buyers, whereas, by the Addition of a *Ladies Oracle*, and other Improvements to the ſaid *Diary*, of late Years, the Sale of it, you KNOW, has been increaſed by ſeveral Thouſands, as the Number had before *ſunk* by being overloaded with the *Problems* and *Solutions* complained of, which being of late too many for that Claſs of Readers, our *Palladium* was forced to ſupply every Reader with their Humour, at the ſame Time every Thing therein is connected with what the *Diary* contains, and they ought to be read together.

The Diary and Palladium are principally ſupported by the *Compiler*'s private Correſpondents, and not by common Contributors, whoſe Productions we inſert to oblige.

Were the Diary only compoſed of ſuch Materials as the Contributors (independent of ourſelves) furniſh, it might be ſuch a one as our ignorant Accuſer would ſuggeſt; but we have no Occaſion to deprive the *Diary* of Materials for the *Palladium*, which we never wanted, but have in Plenty. We are,

Gentlemen, &c.

Χεφαλ μονοφθαλμικος.

P. S. The above Mathematician took his Degree in a Tobacco *Hogſhead* (*See Palladium*, p. 14.) and has taken the Oaths to a proſtitute Writer in Town, as we are informed; and the beſt Writer againſt both is ONE, who ſhall ſign the Warrant for their Execution.

N°. V.

To the Compiler of the Ladies Diary.

SIR,

YOU are certainly unhappy in a bad Memory, otherwiſe it would be impoſſible, for a Man of Honour and Integrity, to fall into ſuch groſs Contradictions. You affirm, in the *Gazetteer* of the 14th Inſtant *, in Anſwer to my Charge againſt you, that *you never wanted Materials to carry on the* Diary *and the* Palladium, *but have in Plenty*: Though, in the *Diary* itſelf, at p. 40, you expreſsly deſire and requeſt that *your Correſpondents will*

* *December.*

will favour you with ſome new, uſeful, and eaſy Queſtions, with their Solutions, in the ſeveral Branches of Science. What an Idea the World muſt entertain of your Veracity, and the Merits of a Cauſe that has forced you upon ſuch palpable Inconſiſtences to ſupport it, is eaſy to determine.

In Anſwer to what I obſerved concerning your propoſing Queſtions in the Mathematicks that are utterly impoſſible and abſurd, you are pleaſed to take Notice, *that the* Ladies Diary *depends more on Amuſement for the Fair Sex than on the Propriety of Mathematical Problems, and that, were my Objections juſtifiable, it would concern but about five Hundred Buyers:* Which I believe may be true. But then, will it not appear ſomething extraordinary to the World, that you, *by whom Errors in the Science of Aſtronomy have been diſcovered and corrected, by whom Mathematical Calculations have been improved, Natural Philoſophy rendered eaſy and uſeful,* &c. &c. and who propoſe *to improve the ingenious Youth of* Britain, *and learn them, by Degrees, to rival the greateſt Maſters of Science** ; I ſay, muſt it not appear very ſtrange, that a Perſon of ſuch prodigious Merit, ſhould fly to ſo *poor* a Plea in his own Defence, and treat it as a Matter of little Conſequence, whether the Problems he publiſhes in the very Science he undertakes to improve, are proper or improper, rational or abſurd.

Give me leave to tell you, Sir, whatever may be the Caſe with Reſpect to the Majority of your Readers, your Recurring to ſo pitiful a Subterfuge ſuits ill with the Character of a Philoſopher and Mathematician, from whom the World is promiſed ſuch Wonders. You think it very hard *to engage in a Diſpute with an unknown Perſon:* But ſurely, of all Men living, you have the leaſt Reaſon to complain

* *See the Preface to the Palladium for* 1751.

plain on that Score. How many Persons of Reputation and superior Genius have you under different, fictitious, Names grossly insulted and abused? Even to that Degree, that the *Diary* under your Hands, is become remarkable for Scandal and Defamation; which are there dealt out with unbounded License.—Had it not been for This you had never been molested by *Honestus*: But to see Men of distinguished Merit, and the most amiable Characters *, treated with such insufferable Insolence by a wretched Pretender, must fill every generous Mind with Indignation, and make them despise the Wretch, whose Faults they might otherwise be inclinable to Pardon.

I am, Sir,

Honestus.

Sir,

BY inserting the inclosed Advertisements you'll oblige a great Number of People, who think themselves very ill used by the Person to whom they are addressed, and, amongst the rest,

Your Correspondent,

Honestus.

* *See* p. 42 *and* 43 *of the* Ladies Diary *for the same Year.*

MATHEMATICAL EXERCISES:

CONTAINING,

I. The Principles of Dialling.

II. The Inveſtigation of the Sums of Series's.

III. Anſwers to the Problems, Paradoxes and Enigmas in the ſecond Number.

IV. A Collection of new Problems.

V. Some Obſervations on the *Ladies Diary*.

By *JOHN TURNER*.

N°. III.

LONDON:

Printed for JAMES MORGAN, at the *Three-Cranes* in *Thames-ſtreet*, 1751, where the preceding Numbers may be had.

ADVERTISEMENT.

ALL Perſons, inclining to encourage this Work, are deſired to ſend their Performances (whether new Problems, Paradoxes, Solutions, *&c.*) Poſt paid, to be left with Mr. *James Morgan*, at the *Three-Cranes*, in *Thames-ſtreet*, before the 20th of *April*, 1752.

The Fourth Number being to be publiſhed on the 1ſt of *June* enſuing.

ANSWERS to the PROBLEMS propoſed in the SECOND NUMBER.

PROBLEM XVII. *Anſwered by the Propoſer Mr.* Marſhall.

PUT x for the greater Number and y for the leſſer One; then, by the Problem, $48+x : 48+y :: 6 : 5$, and $48-x : 48-y :: 2 : 3$; therefore $240+5x=288+6y$ and $144-3x=96-2y$; whence x is found $=24$ and $y=12$.

The ſame anſwered by Mr. P—k George *of the* Iſle of Wight.

Let x repreſent the greater Number and y the leſſer One; then, 48 being denoted by b, we have, by the Problem, $b+y : b+x :: 5 : 6$, therefore $6b+6y=5b+5x$: In like Manner $b-y : b-x :: 3 : 2$, therefore $2b-2y=3b-3x$; from whence and the preceding Equation $2x=b$; conſequently $x=24$ and $y=12$.

This Problem was, alſo, anſwered by Mr. *R—ſon*, Mr. *Widd*, *Bideſordienſis*, Mr. *Phipps*, and *L. N——n*.

PROBLEM XVIII. *Anſwered by Mr.* W. Phipps.

Let the Area of the Triangle be denoted by A, the given Difference by a, and let the Perpendicular, falling from the Right-Angle upon the Hypothenuſe, be denoted by x: Then will the Hypothenuſe be expreſſed by $a+x$; therefore $a+x \times$

$\frac{ax}{2} = A$, which reduced gives $x = \frac{\sqrt{8A+a^2}-a}{2}$; from whence the Sides and Angles may be found.

The ſame anſwered by Bidefordienſis.

Put a to expreſs the propoſed Area, d the given Difference and x the Hypothenuſe; then will $x^2 - xd = 2a$, whence $x = \frac{d+\sqrt{8a+d^2}}{2}$. But it is known that the Baſe of any Triangle is to twice the Perpendicular as the Sine of the Vertical Angle is to a fourth Number, whoſe Difference from the Verſed Sine of the Supplement of the ſaid Angle is equal to the Verſed Sine of the Difference of the Angles at the Baſe; therefore, in this Caſe, it will be as $x : 2x-2d :: 1 : \frac{2x-2d}{x}$, which taken from 1 leaves $1 - \frac{2x-2d}{x}$ for the Verſed Sine of the Difference of the Angles at the Baſe; from whence the Angles themſelves and conſequently the Sides may be found.

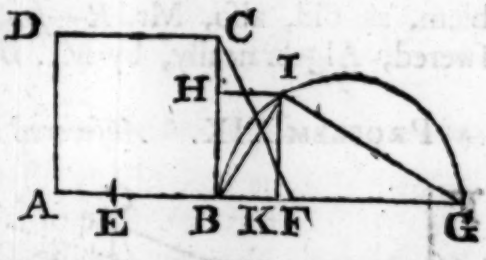
The ſame anſwered by Mr. George.

Let the given Area of the Triangle be expreſſed by a, the given Difference by $2n$, and the Sum of the Hypothenuſe and Perpendicular by $2x$; then will the Hypothenuſe be denoted by $x+n$, and the Perpendicular by $x-n$, therefore $x^2 - n^2 = 2a$, whence x will be had $= \sqrt{2a+n^2}$. Let now $2m$ be put for the Hypothenuſe q for the Perpendicular and $2z$ for the Difference of the Segments of the Hypothenuſe, then $m+z$ will expreſs the greater Segment and $m-z$ the leſſer One; therefore, by the Nature of the Circle, $m^2 - z^2 = q^2$ and $z = \sqrt{m^2-q^2}$; from whence the Sides and Angles may be determined.

The

The ſame anſwered, Geometrically, by the Propoſer Mr. Moſs, *who alſo ſent an Algebraic Solution.*

CONSTRUCTION.



Let the Square AC be conſtructed ſo that the Area thereof may be equal to twice the given Area, and in AB, produced, take BF equal to Half the propoſed Difference between the Hypothenuſe and Perpendicular; join F, C and take FE, FG, each, equal to FC; then having deſcribed a Semicircle on BG make BH=BE, and parallel to AB draw HI meeting the Semicircle in I; join I, B and I, G and BIG will be the Triangle required.

DEMONSTRATION.

Draw IK perpendicular to AG.

Becauſe FE—EB=FB= Half the given Difference, it follows that FG—EB will likewiſe be equal to Half the given Difference and conſequently GB—IK (EB) = the whole Difference. Moreover, ſince FE=FC=FG, by Conſtruction, it is evident that a Semicircle, deſcribed on EG, will paſs thro' the Point C; therefore CB and IK being each perpendicular to AG, and IK=EB (BH) by Conſtruction, it will appear that GB × (EB) IK= BC^2= twice the given Area.

Q. E. D.

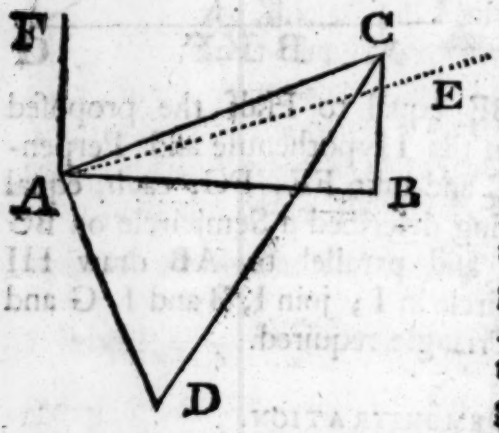
Method of Calculation.

In the Right-angled Triangle BFC are given BF and BC, whence FC and conſequently IK=

FE (FC) — FB and GB = FC + FB may be had; Then, by the Property of the Circle, $BI = \sqrt{GB \times BK}$ and $GI = \sqrt{GB \times GK}$; whence the Angles may be found.

Mr. *L. L—n* ſent a Conſtruction to this Problem, as did, alſo, Mr. *R—ſon*, and it was anſwered, Algebraically, by Mr. *Widd.*

PROBLEM XIX. *Anſwered by Mr.* Widd.



In the Triangle ABC are given AB (= 14) and BC (= 6,) by which the Angle BAC, expreſſing the Courſe made good, and the Side AC, expreſſing the Diſtance made good, are found equal to 23° 12′ and 15.23 Miles: Then in the Triangle ADC are given the three Sides, by which the Angles CAD, ACD and ADC are found equal to 70° 50′, 45° 7′ and 64° 3′ reſpectively; whence the Ship's Courſe on her Larboard Tack will be had equal to 42° 22′ from the South towards the Eaſt, ſhe lying at that Time 70° 8′ from the Wind, and That on her Starboard Tack, ſhe lying then 45° 49′ from the Wind, 21° 41′ from the North towards the Eaſt.

The ſame anſwered by the Propoſer Nauticus.

Let AF be the Meridian paſſing thro' (A) the Port from whence the Ship departs; make AB perpendicular and BC parallel to AF, the former equal to the given Departure, and the latter equal to the

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The proposer of Prob 19 N°2

seems not rightly to understand

the nature of the Subject; for he

tacitly supposes the Ship not to

make the same angle with the

Wind on each Tack (as is manifest

from his Solution) but as that is

universally allowed to be the

Case therefore the problem as it

stands proposed is absurd or

at least over limited. But if

the departure be omitted in the

data the problem will be

truly limited.

the Difference of Latitude made good; join A, C upon which let the Triangle ADC be conſtituted ſo that AD be equal to 12, and CD equal to 16; alſo make the Angle FAE equal to the Angle which the Direction of the Wind makes with the Meridian: Then in the right-angled Triangle ABC are given the two Legs, whence AC will be found = 15.23 and the Angle BAC = 23° 12′, conſequently CAE = 42′; alſo in the Triangle CAD will be given all the Sides, whence the Angle CAD will be had = 70° 53′, DCA = 45° 7′ and ADC = 64°; but DAC — BAC = 47° 41′ = the Complement of 42° 19′ the Angle which the Ship makes with the Meridian on her Larboard Tack towards the Eaſt, ſhe then lying 70° 11′ from the Wind, and ACB — ACD = 21° 41′ the Angle which ſhe makes on her Starboard Tack from the Meridian Eaſterly, lying at that Time 45° 49′ from the Wind.

Meſſ. *Moſs*, *R—ſon*, and *N—n*, ſent their Methods of anſwering this Problem.

PROBLEM XX. *Anſwered by Mr.* Phips.

Divide the whole Expreſſion, $x^3 - y^3 = 2 \times \overline{x^2 - y^2} = 3 \times \overline{x - y}$, by $x - y$ and you will have $x^2 + xy + y^2 = 3$ and $2 \times \overline{x + y} = 3$, or $y = \frac{3 - 2x}{2}$. Subſtitute this Value of y and alſo its Square in the Equation $x^2 + xy + y^2 = 3$, then reduce it and it will become $x^2 - \frac{1}{2}x = \frac{1}{4}$; which ſolved gives $x = \frac{3 + \sqrt{21}}{4} = 1.8955$ nearly: Hence, $2x$ being greater than 3, it appears that y will have a Negative Value.

The

The ſame anſwered by Mr. L. N——n *of* Camberwell.

If the given Expreſſion be divided by $x-y$, it will become $x^2 + xy + y^2 = 2x + 2y = 3$, therefore $y = \frac{3-2x}{2} =$ (by putting $\frac{3}{2} = a$) $a-x$, and $x^2 + xy + y^2 = 2a$, or $a^2 - ax + x^2 = 2a$; whence $x = \frac{a}{2} + \frac{\sqrt{8a-3a^2}}{2} = \frac{3+\sqrt{21}}{4}$, and $y = \frac{a}{2} - \frac{\sqrt{8a-3a^2}}{2} = \frac{3-\sqrt{21}}{4}$.

The ſame anſwered by Mr. T. Leigh.

Given $x^3 - y^3 = 2x^2 - 2y^2 = 3x - 3y$; from whence the Expreſſion $x^2 + xy + y^2 = 2x - 2y = 3$ is derived, and from thence $x = \frac{3+\sqrt{21}}{4}$.

This Problem was, alſo, anſwered by Meſſ. *Graham*, *R—ſon* and *George*. Beſides the above Methods of Solution, the Problem will admit of another wherein $x = y$; in which Caſe, x and y being alike concerned with different Signs, each Part of the Equation will become $= 0$.

PROBLEM XXI. *Anſwered by Mr.* R.R.H. *of* Southwark.

Let a, b, c, d, e, f, g and h be the Quotients made by dividing the Leaſt of the three required Numbers by the given Diviſors 2, 3, 4, 5, 6, 7, 8 and 9 reſpectively, and let x denote that Number; then by the Problem $x = 2a + 1 = 3b + 2 = 4c + 1 = 5d + 4 = 6e + 5 = 7f + 1 = 8g + 5 = 9h + 2$. Therefore, comparing the two firſt Values of x together, we have $a = b + \frac{}{b}$

$\frac{b+1}{2}$ which ought to be a whole Number; therefore $\frac{b+1}{2}$ is a whole Number, which put $= n$, then $b = 2n - 1$ and $3b+2 = 4c + 1 = 6n - 1$ therefore $c = n - 1 + \frac{n+1}{2}$ and $\frac{n+1}{2}$ a whole Number which put $= m$; then take the Value of x in Terms of m and compute the Value of d, alſo put another Letter for the Fractional Part of that Value, and ſo proceed with all the Terms, and you will at laſt have $x = 2520\,r - 2491$, which if r be taken equal to 1 will become $x = 2520 - 2491 = 29$. Now if 29 be added to 2520 which is the leaſt common Multiple of all the Diviſors, the Sum 2549 will be the ſecond Number required, and if to this 2520 be added their Sum 5069 will be the third Number ſought.

After the very ſame Manner, this Problem was anſwered by Mr. *J. H*———*n* the Propoſer and by Mr. *N*——*n*.

The ſame anſwered by Mr. George.

Let x repreſent the required Numbers, then $\frac{x-2}{9}$, $\frac{x-5}{8}$, $\frac{x-1}{7}$, $\frac{x-5}{6}$, $\frac{x-4}{5}$, $\frac{x-1}{4}$, $\frac{x-2}{3}$, and $\frac{x-1}{2}$ muſt be all whole Numbers: Let therefore $\frac{x-2}{9}$ be put $= n$, and x will be found $= 9n+2$; ſubſtitute this in the ſecond Expreſſion for x and it will become $\frac{9n-3}{8} = n + \frac{n-3}{8}$; but $\frac{n-3}{8}$ is to be a whole Number; let it therefore be put $a =$ and we ſhall have $n = 8a + 3$, which ſubſtituted for n gives $x = 72a + 29$, whence the third Expreſſion becomes $\frac{72a+28}{7} = 10a + 4 + \frac{2a}{7}$: Put $m = \frac{2a}{7}$, then

then $a = \frac{7m}{2}$, $x = 252m + 29$, and the fourth Expreſſion $\frac{252m+24}{6} = 42m + 4$: Now, as this laſt Expreſſion is a whole Number, let the Value of x laſt found be ſubſtituted in the fifth Expreſſion and it will become $\frac{252m+25}{5} = 50m + 5 + \frac{2m}{5}$; put $s = \frac{2m}{5}$, then $m = \frac{5s}{2}$, $x = 630s + 29$ and the ſixth Expreſſion $\frac{630s+28}{4} = 157s + 7 + \frac{s}{2}$: Let $\frac{s}{2} = v$, then $s = 2v$, $x = 1260v + 29$ and the ſeventh Expreſſion $\frac{1260v+27}{3} = 420v + 29$, in which there is no Remainder; therefore the Value of x laſt found being ſubſtituted in the laſt Expreſſion it becomes $\frac{1260v+28}{2} = 630v + 14$ where there is alſo no Remainder; whence it appears that $1260v + 29$ will expreſs the general Value of x, v being taken, either, $= 0$, or ſome Affirmative even whole Number. For as 2×1260 is $= 2520$ the leaſt common Multiple of all the Diviſors, it is evident that when $v = 0$, $x = 29$; when $v = 2$, $x = 2549$ and when $v = 4$, $x = 5069$ which is the third Number ſought.

Meſſ. *R—ſon*, *Graham*, and *Widd* alſo anſwered this Problem.

PROBLEM XXII. *Anſwered by Mr.* Graham.

Let x be the Height required, then ſince the Velocities of falling Bodies are as the Square Roots of the Diſtances perpendicularly deſcended, the abſolute Celerity of the Ball after it has deſcended thro' the Height x will be as $\sqrt{x}$, and its Momentum as

$4\sqrt{x}$. Now if c be put for the Coſine of 40°, the Inclination of the Plane that receives the Impulſe, Radius being Unity, it will be as $1 : c :: 4\sqrt{x} : 4c\sqrt{x}$ the Force with which the Ball impinges on the ſaid Plane. In like Manner the Force impreſſed on the other Plane by the Ball of 9 Ounces will be found to be $9n\sqrt{12}$, where n is put for the Coſine of 25° the Inclination of the ſaid Plane to the Horizon: Whence we have this Equation $4c\sqrt{x} = 9n\sqrt{12}$, which being reduced gives $x = \frac{243n^2}{4c^2} = 85$ Feet nearly.

The ſame anſwered by Mr. Widd.

Suppoſe the Force of the greater Ball, falling from the Height of 12 Feet, on the Plane whoſe Inclination is 25° to be 1; then will the Impetus of the leſſer Ball, falling from the Height of 12 Feet, on this Plane be $= \frac{4}{9}$: But by the Laws of Mechanics Sine of 65° : Sine 50° : : $\frac{4}{9}$: .37566 = the Impetus of this Ball, from the given Height, on the Plane whoſe Inclination is 40°. Now, the Force being as the Velocity, and the Velocity as the Square Root of the Space deſcended thro', to determine the Height a Ball muſt fall from to gain an Impetus = 1 that acquires an Impetus = .37566 in falling 12 Feet, ſay as $\overline{.37566}|^2 : 1^2 :: 12 :$ 85.034 Feet the Height required.

The ſame anſwered by Mr. R———ſon.

Let x be the required Height; then ſince the Velocity is as the Square Root of the Diſtance perpendicularly deſcended, we ſhall have $9\sqrt{12}$ for the Force of the Ball of 9 Ounces and $4\sqrt{x}$ for the Force of the Ball of 4 Ounces: But, by the

Principles of Mechanics, the abſolute Force of a Body is to the Force exerted on a given Plane, as Radius to the Coſine of the Planes Inclination to the Horizon; therefore, putting a for the Sine of 65°, and b for the Sine of 50°, we have $1 : a :: 9\sqrt{12} : 9a\sqrt{12}$, the Force exerted on the Plane whoſe Inclination is 25°, and $1 : b :: 4\sqrt{x} : 4b\sqrt{x}$, the Force exerted on the Plane whoſe Inclination is 40°; whence, by the Problem, $4b\sqrt{x} = 9a\sqrt{12}$, and $x = \frac{243a^2}{4b^2} = 85.033$ the Height required.

This Problem was alſo anſwered by Mr. *Moſs*.

PROBLEM XXIII. *Anſwered by Mr.* Graham, *the Propoſer.*

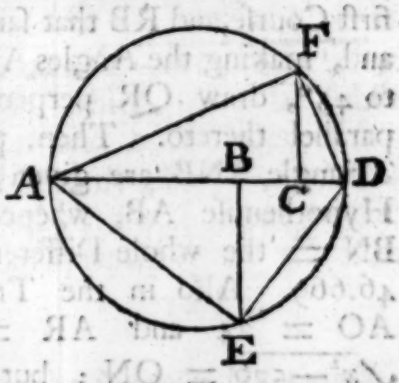
Let the Sines be to each other as m to n, and the Coſines as p to q, then if the Sines be denoted by x and y, we ſhall have $x : y :: m : n$, and $\sqrt{1-x^2} : \sqrt{1-y^2} :: p : q$ Whence, by Squaring the Terms of the laſt Proportion and ſubſtituting inſtead of x its Value $\left(\frac{ym}{n}\right)$ derived from the firſt, we have $1 - \frac{m^2y^2}{n^2} : 1-y^2 :: p^2 : q^2$, conſequently $q^2 - \frac{q^2m^2y^2}{n^2} = p^2 - p^2y^2$ and $y = \sqrt{\frac{q^2-p^2}{\frac{m^2q^2}{n^2} - p^2}}$; whence every Thing required may be had.

The ſame anſwered by Mr. Leigh.

Let the Sines be denoted by x and y, and let their Ratio be that of 1 to a, alſo let the Ratio of the Coſines be that of 1 to c; then will $a^2x^2 = y^2$ and $c^2 \times \sqrt{1-x^2} = 1-y^2$; whence $2x^2 =$

$\frac{2x\sqrt{1-c^2}}{a^2+c^2}$ = the Verſed Sine of twice the Arc whereof x is the Sine to Radius, Unity.

The ſame anſwered by Mr. R——ſon.



Let m and n repreſent the given Ratio of the Sines, and p and q the given Ratio of the Coſines; take any Line DB which divide in C ſo that DC : DB : : m^2 : n^2, and produce DB till AB : AC : : p^2 : q^2; then upon the Diameter AD let a Circle be deſcribed, and let the Perpendiculars BE, CF be drawn to meet the Periphery thereof; join A, E; E, D; A, F and F, D and the Thing is done.

DEMONSTRATION.

By the Property of the Circle AD × CD = $\overline{DF}|^2$, AD × BD = $\overline{DE}|^2$, AD × AC = $\overline{AF}|^2$ and AD × AB = $\overline{AE}|^2$; therefore $\overline{DF}|^2$: $\overline{DE}|^2$: : AD × CD : AD × BD : : CD : BD : : by Conſtruction, m^2 : n^2, and $\overline{AF}|^2$: $\overline{AE}|^2$: : AD × AC : AD × AB : : AC : AB : : by Conſtruction, q^2 : p^2; whence DF : DE : : m : n and AF : AE : : q : p.

Q. E. D.

Mr. *Moſs* ſent a Geometrical Conſtruction to this Problem, and *Bideſordienſis*, Mr. *Widd* and Mr. *N—n* ſent Algebraic Solutions to it.

PROBLEM XXIV. *Anſwered by Mr.* Widd.

In the following Scheme let A repreſent the Place the Ship ſailed from, and B the Port ſhe arrived at; alſo let AR be the Diſtance ſailed on the firſt Courſe, and RB that ſailed on the ſecond Courſe, and, making the Angles ABN and BAN each equal to 45°, draw QR perpendicular to BN and RO parallel thereto. Then, putting AR = x, in the Triangle ANB are given all the Angles and the Hypothenuſe AB, whence the whole Departure BN = the whole Difference of Latitude AN = 46.669. Alſo in the Triangle ARO are given AO = 24 and AR = x, whence RO = $\sqrt{x^2-576}$ = QN; but QN + BN = BQ, therefore $\overline{QR|}^2+\overline{QB|}^2=\overline{BR|}^2$, *that is* $2697.879+93.3385\sqrt{x^2-576}+x^2-576=6084-156x+x^2$; which reduced gives $x = 24.1033$ Miles: Hence RB = 53.8967, the firſt Courſe 5° 18′ from the North towards the Weſt and the ſecond Courſe 65° 8′ from the North towards the Eaſt.

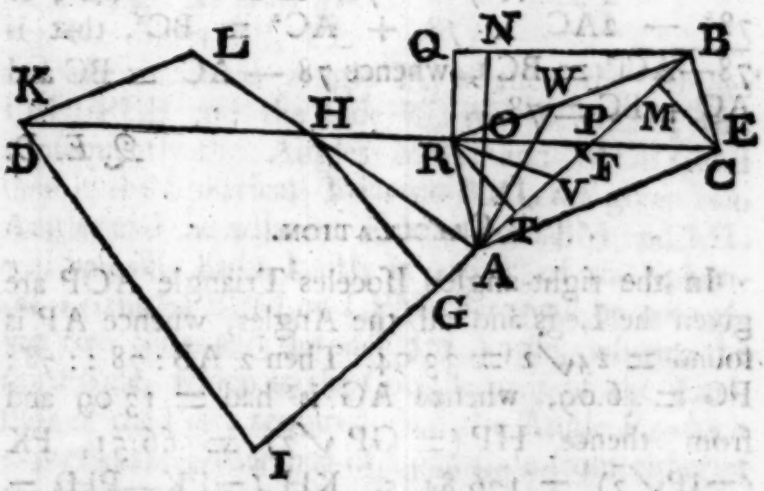
The ſame anſwered by Mr. Moſs.

Let A, in the ſubſequent Scheme, be the Place the Ship ſailed from, B the Port ſhe was bound to, and AR, RB the Diſtances ſhe ſailed on each Courſe: Alſo let AN be the Difference of Latitude, which, in this Caſe, is equal to (BN) the Departure when the Ship arrives at the Port B. Draw RT perpendicular to AB and take TV = AT and RW = AR; alſo join R, V and A, W, and draw RO parallel to BN; then will NO (= AN — AO) be known, and from thence PB and PA may be found. Now, if AB be denoted by a, AP by b, AR + RB by c, the Sine of 45° by n, and RP by x (Radius being Unity) we ſhall have

have $1 : x :: n : nx = TP = RT$; therefore AT (TV) $= b - nx$, BV $= a - 2b + 2nx$, and AR (=RV) $= \sqrt{AT|^2 + RT|^2} = \sqrt{b^2 - 2bnx + 2n^2x^2}$; alſo WB(=RB—RA) $= c - 2\sqrt{b^2 - 2bnx + 2n^2x^2}$: Whence $a : c :: c - 2\sqrt{b^2 - 2bnx + 2n^2x^2} : a - 2b + 2nx$, and $a^2 - 2ba + 2anx = c^2 - 2c\sqrt{b^2 - 2bnx + 2n^2x^2}$, or $2c\sqrt{b^2 - 2bnx + 2n^2x^2} = c^2 + 2ab - a^2 - 2anx$, which, if d be ſubſtituted for $c^2 + 2ab - a^2$ and both Sides of the Equation ſquared, will become $4b^2c^2 - 8bc^2nx + 8c^2n^2x^2 = d^2 - 4adnx + 4a^2n^2x^2$; from whence x may be found, and conſequently what elſe is required.

The ſame anſwered by the Propoſer Nauticus.

CONSTRUCTION.



Let AN be the Meridian paſſing thro' (A) the Port where the Ship departed from, and make the Angle NAB=45°; take AO=24 and AB=66; biſect AB in F and take FG a Third-Proportional to 2AB and 78; alſo take GI = 78 and having drawn DOE perpendicular to AN draw GH and IK,

IK, perpendicular to AB, meeting DE in H and K; join H, A and from the Center K, with the Interval AB, let an Arch be described cutting HA, produced if need be, in L; draw LK and AC, parallel to it, meeting DE in C; lastly join B, C and the Thing is done.

Demonstration.

Let CM be perpendicular to AB: Then, because of the Parallel Lines, KL (AB) : AC (:: KH ; CH) : : GI (78) : GM, and consequently AB × GM = AC × 78. Moreover, (*by Construction*) 2AB : 78 : : 78 : FG; therefore AB × FG = $\frac{1}{2}\overline{78|^2}$ which, taken from AB × GM = AC × 78, leaves AB × FM = AC × 78 — $\frac{1}{2}\overline{78|^2}$ and therefore 2AB × FM = 2AC × 78 — $\overline{78|^2}$. But 2AB × FM is also = $AC^2 - BC^2$, therefore 2AC × 78 — $\overline{78|^2} = AC^2 - BC^2$, or 78^2 — 2AC × 78 + $AC^2 = BC^2$, that is $\overline{78-AC|^2} = BC^2$, whence 78 — AC = BC and AC + BC = 78.

Q. E. D.

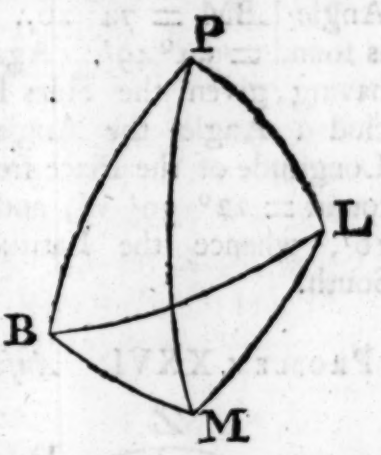
Calculation.

In the right-angled Isoceles Triangle AOP are given the Legs and all the Angles, whence AP is found = $24\sqrt{2} = 33.94$. Then 2 AB : 78 : : 78 : FG = 46.09, whence AG is had = 13.09 and from thence HP (= GP $\sqrt{2}$) = 66.51, PK (= IP $\sqrt{2}$) = 176.8185, KH (= PK—PH) = 110.3 and HO (= HP—OP) = 42.51. Moreover OP : Tang. 45° : : HO : Tang. HAO which is 60° 33′, and KL : *S.* KHL (Cof. HAO) : : KH : *S.* KLH (= *S* HAC) which is 124° 45′ whence OAC and BAC will each be had. Therefore

fore *S.*OCA : OA : : Rad. : AC=55.143 and BC (=78—AC) :*S.*45° :: PB : *S.*PCB which is 82° 39′.

Mr. *R*———*ſon* ſent a Method of conſtructing this Problem, and Meſſ. *Phips* and *George* ſent Algebraic Solutions to it.

PROBLEM XXV. *Anſwered by Mr.* R———ſon.



Let P repreſent the North Pole, L the *Lizard*, B *Barbadoes* and M the Place where the Ships met; alſo ſuppoſe BL to be an Arc of a Great Circle paſſing thro' B and L and PM part of the Meridian paſſing thro' M; then in the Spherical Triangle BPL are given the Sides BP, PL and the included Angle P, whence the Angles PBL, PLB and the Side BL may be found, and conſequently the Angles MBL and MLB; and then in the Spherical Triangle BML are given two Angles and the adjacent Side whence BM and ML will be eaſily had: Laſtly in either of the Spherical Triangles BPM or LPM there will be *now* given two Sides and the included Angle, whence the third Side, which is the Complement of the Latitude of the Place required, and the Angle P, which expreſſes the Difference of Longitude from either of the Ports, according to the Triangle you calculate from, may be found.

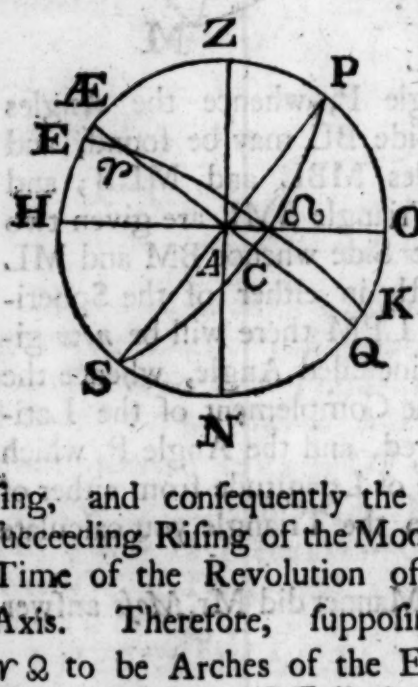
After the very ſame Manner did Mr. *Moſs* anſwer this Problem.

The

The same answered by Mr. Widd.

In the Spherical Triangle PLB are given the two Sides PL, PB with their included Angle, whence the Side BL will be found = 57° 7′, the Angle PLB = 110° 48′ and the Angle PBL = 38° 4′. Then in the Triangle BLM are given the Side BL, the Angle BLM = 41° 4′ and the Angle LBM = 74° 26′, by which the Side LM is found = 54° 49′. Again in the Triangle PLM, having given the Sides PL, LM with their included Angle, the Angle MPL, expressing the Longitude of the Place from the Lizard, will be found = 22° 39′ W, and the Side PM = 91° 16′, whence the Latitude required is 1° 16′ South.

PROBLEM XXVI. *Answered by Mr.* L. N—n.



It appears that the Latitude of the Place sought will be equal to the Complement of the Moon's greatest Declination. For then the Orbit of the Moon will coincide with the Horizon of the Place, at the Time of the Moon's Rising, and consequently the Time between each succeeding Rising of the Moon will be equal to the Time of the Revolution of the Earth about its Axis. Therefore, supposing A♈, A♌, and ♈♌ to be Arches of the Equator, Horizon (or Moon's Orbit) and Ecliptic respectively, in the Triangle A♈♌ are given the two Angles ♈, ♌ the

with the adjacent Side, whence the Angle A will be found and alſo its Supplement which is = 27° 15′ the Complement of the Latitude required.

After the very ſame Manner RRH anſwered this Problem.

The ſame anſwered by Mr. Widd.

In the Caſe propoſed the Moon's Orbit will coincide with the Horizon: Let, therefore, OAH repreſent a Part of the Horizon or Lunar Orbit, ♈A a Part of the Equator and ♈♋ a Part of the Ecliptic; then in the Triangle A♈♋ are given ♈♋ = 45°, the Angle ♈♋A = 5° and the Angle A♈♋ = 23° 29′, whence the Angle A is found = 152° 45′; the Exceſs (62° 45′) of which above 90° is the required Latitude North.

PROBLEM XXVII. *Anſwered by the Propoſer Mr.* Leigh.

Let 1.0017523, the Amount of 1*l.* and its Intereſt for 1 Week, be denoted by r, 365 the Weeks in 7 Years by n, the given Sum by a, and the Allowance required by x; then the Amount of a in 1 Week will be ra and, deducting the Allowance, $ra-x$ will be the Sum put out the 2d Week; the Amount of which will be r^2a-rx. In like Manner, after deducting the Weekly Allowance, the Amount of the Sum put out the 3d Week will be r^3a-r^2x-rx; whence we have $a \times r^n-x \times \overline{r^{n-1}+r^{n-2}+r^{n-3} \cdots\cdots r^2+r+1} = 0$: But, by Prob. IV. in the Summation of Series N 2, $r^{n-1}+r^{n-2}+r^{n-3} \cdots\cdots r^2+r+1$ is equal to $\frac{1-r^n}{1-r}$, therefore $ar^n-x \times \frac{1-r^n}{1-r} = 0$, conſequently

$x = \frac{r^{n+1} - r^n}{r^n - 1} \times a = 6l.\ 5s.\ 4d.$ the Allowance required.

The ſame anſwered by Mr. Graham.

Let x denote the Weekly Allowance and m the Amount of 1 Pound in 1 Week; then $\frac{x}{m}$ will denote the preſent Worth of x Pounds to be received a Week hence and $\frac{x}{m^2}$ the preſent Worth of the ſame to be received two Weeks hence, &c. conſequently the preſent Worth of all the Weekly Allowances will be defined by $\frac{x}{m} + \frac{x}{m^2} + \frac{x}{m^3} + \text{-------} \frac{x}{m^{365}}$: Which Series muſt, it is evident, be equal to 2000, or $x \times \overline{\frac{1}{m} + \frac{1}{m^2} + \frac{1}{m^3} + \text{-------} \frac{1}{m^{365}}}^{1} = 2000$; whence $x = \frac{2000 \times \overline{m-1}}{1 - \frac{1}{m^{365}}} = 6l.$ nearly.

Mr. *R*——*n* and Mr. *Widd*, alſo, ſent Anſwers to this Problem.

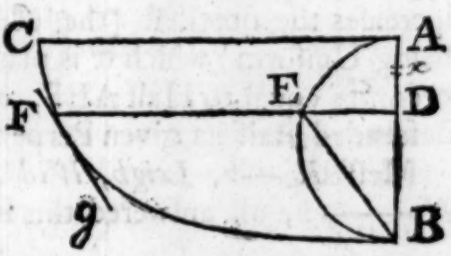
PROBLEM XXVIII. *Anſwered by* ΣΟΦΟΣ.

Let $w^4 - Aw^3 + Bw^2 - Cw + D = 0$ be an Equation whoſe four Roots expreſs the Values of the four required Quantities x, y, z and u; then if the given Quantities 24, 178, 1476 and 13042 be denoted by a, b, c and d reſpectively, the Value of A will be expreſſed by a (24,) the Value of B by $\frac{a^2 - b}{2}$ (199,) that of C by $\frac{c - ab + aB}{3}$ (= 660) and that of D by $\frac{ac - bB + aC - d}{4}$ (= 700.) Therefore our aſſumed

aſſumed Equation, by ſubſtituting theſe Values, is reduced to $w^4 - 24w^3 + 199w^2 - 660w + 700 = 0$; wherein the Roots or Values of w will, by the Method of Diviſors, be found to be 2, 5, 7 and 10, which are the Numbers ſought.

Mr. *R——ſon* ſent an Algebraic Solution to this Problem.

PROBLEM XXIX. *Anſwered by Mr.* Graham.



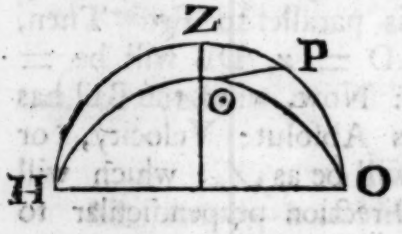
Let ACB be Half the given Cycloid, and let AEB be Half the generating Circle, whoſe Diameter (being the Axis of the Cycloid) is perpendicular to the Horizon: Let the Tangent Fg and the Ordinate FD be drawn, the latter inſecting the Circle in E; alſo join E, B, which by the Nature of the Curve is parallel to Fg. Then, putting AB $= a$ and AD $= x$, DB will be $= a - x$ and EB $= \sqrt{a^2 - ax}$: Now, when the Ball has deſcended thro' CF, its Abſolute Velocity, or that in the Direction Fg will be as $\sqrt{x}$, which will be to its Velocity in a Direction perpendicular to the Horizon as EB to DB: Whence we have $\sqrt{a^2 - ax} : a - x :: \sqrt{x} : \frac{\sqrt{ax - x^2}}{\sqrt{a}}$ the Celerity with which the Ball approaches the Horizon, which by the Problem is to be a Maximum; therefore, as $\sqrt{a}$ is conſtant and $\sqrt{ax - x^2}$ expreſſes the ordinate of the generating Circle, the Velocity required will be a Maximum when the Ordinate becomes equal to the Radius, or when AD becomes equal to $\frac{AB}{2}$.

The ſame anſwered by the Propoſer Mr. Moſs.

Let CB be half the Periphery of the given Cycloid, whoſe Axis is AB, and let AEB be Half the generating Circle; alſo let the ordinate FD be drawn: Then, it being known that the Time of Deſcent thro' CF is as the correſponding Arch AE of the Semicircle, it is evident that the Ball will approach the faſteſt towards the Horizon when AD increaſes the quickeſt (the Motion of the Point E being Uniform) which it is plain muſt be when AE becomes equal to Half AEB, or when the Ball has deſcended Half its given Perpendicular Diſtance.

Meſſ *R*——*n*, *Leigh*, *Widd*, *R. R. H.* and *L. N*——*n*, all anſwered this Problem.

PROBLEM XXX. *Anſwered by the Propoſer* Curioſus.



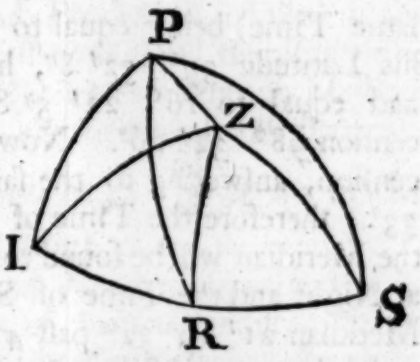
Since the Altitude of the Pyramid and the Side of its Baſe are given, the Inclination of the Slant Side to the Horizon will be found to be 54° 14′, which being leſs than the Complement of the Latitude ſhews the Sun's Declination (6° 6′) to be South when the Pyramid firſt begins to appear without a Shadow. But, in order to find the Time of its continuing without a Shadow on the longeſt Day, let HZO repreſent the Plane of the Meridian, HO that of the Horizon, H⊙O the Plane of that Side of the Pyramid fronting the Weſt Point and P the Pole: Let ⊙P be an Arch of a Great Circle paſſing thro' the Pole and the Point ⊙ where the Sun firſt begins to appear on the Plane; then in the Triangle OP⊙ will be given PO the propoſed Latitude,

P

P⊙ the Complement of the Sun's Declination and the Angle ⊙OP the Complement of the Planes Inclinatien to the Horizon, whence the Angle ZP⊙ will be had = 39° 36′; the Double of which converted into Time gives 5ʰ : 16ᴹ : 48ˢ the Time of the Pyramids continuing without a Shadow.

This Problem was, alſo, anſwered by Meſſ. *Moſs*, *R*———*ſon*, and *Widd.*

PROBLEM XXXI. *Anſwered by Mr.* R———ſon.



Let P repreſent the North Pole, Z the Zenith of the Place where the Obſervation was made, alſo S and I the two Stars at the Time propoſed. Put *s* and *c* for the Sine and Coſine of ZP, *a* and *b* for the Sine and Coſine of SP, *f* and *g* for the Sine and Coſine of IP and *m* and *n* for the Sine and Coſine of IPS (the Difference of the Right Aſcenſion of the two Stars) all which are eaſily found; alſo let *x* and *y* denote the Sine and Coſine of the Angle ZPI, then $my-nx$ and $ny+mx$ will be the Sine and Coſine of the Angle ZPS; whence (by Spherics and the Nature of the Problem) we ſhall have $sa \times \overline{ny+mx} + bc = sfy + gc$, and $y^2 + 2dxy + d^2x^2 = p^2 \times \overline{x^2+y^2}$, where $d = \frac{ma}{an-f}$ and $p = \frac{gc-bc}{san-sf}$; conſequently $\frac{x}{y} = \frac{\sqrt{\overline{d^2+p^2-1} \times \overline{d^2-p^2}^2}}{d^2-p^2} - \frac{d}{d^2-p^2}$ = the Tangent of the Angle ZPI; which being

being had the Hour of the Night will be eaſily found.

The ſame anſwered by the Propoſer Curioſus.

The Geocentric Longitude of Jupiter (at Midnight between the two given Days) being found, by the Tables, equal to 19° 23′ 3″ of Piſces, and the Latitude correſponding 1° 31′ 7″ South; the Declination will from thence be found equal to 5° 37′ 51″ and the Right Aſcenſion 350° 51′ 26″: In like Manner, the Longitude of Sirius (at the ſame Time) being equal to 10° 38′ 50″ of ♋ and his Latitude 39° 32′ 8″, his Declination will be had equal to 16° 22′ 5″ South, and Right Aſcenſion 98° 32′ 26″. Now the Sun's Right Aſcenſion, anſwering to the ſaid Time, is 211° 27′ 33″; therefore the Time of Jupiter's coming on the Meridian will be found to be 17′ 35″ 31‴ paſt 9 at Night and the Time of Sirius's coming on the Meridian 21′ 19″ 32‴ paſt 4 in the Morning.

In the annexed Scheme, let P repreſent the Pole, Z the Zenith of the given Place, I the Place of Jupiter and S the Place of Sirius at the Time of Obſervation; then, if ZI, ZS and IS, alſo ZR, perpendicular to IS, and PR be drawn, it is evident that IR muſt be equal to RS: Therefore, in the Triangle IPS, the two Sides IP, PS with their included Angle being given, the Side IS and the Angles PSI, PIS will be found: Then in the Triangle PRS the two Sides PS, SR and their included Angle are given, whence PR and the Angles PRS, RPS will be had. Again in the Triangle PRZ are given the two Sides RP, PZ and the Angle PRZ (=PRS—90°) whence the Angles RPZ, PZR and the Side ZR will be found. But as RPS — RPZ = SPZ = 44° 30′ 37″ is the Number of Degrees which Sirius wants of the Meridian

ridian at the Time of Obſervation, this converted into Time gives $\cdot 2^{H}\ 58^{M}\ 2^{S}\ 24^{III}$ for the Time that will be taken before he appears upon the Meridian: Hence it is evident that the Time of Obſervation was $1^{H}\ 30^{M}\ 17^{S}\ 8^{III}$ paſt Midnight.

Mr. *Widd* anſwered this Problem and found the Time to be $1^{H}\ 30^{M}\ 16^{S}$ paſt Midnight.

PROBLEM XXXII. *Anſwered by Mr.* Graham.

The Beam, deſcending with the two unequal Weights fixed to the Ends thereof, may be conſidered as a Compound Pendulum, oſcillating with the greateſt lateral Oſcillation; and therefore, when reduced to a Simple One by the common Methods, the Diſtance between the Center of Oſcillation and Point of Suſpenſion will be $= \frac{d+b}{d-b} \times a$; in which a denotes Half the Length of the Beam and d and b the 18 and 12 Pound Weights reſpectively. But the Time of Deſcent of ſuch a Pendulum is equal to That in which a Body might freely deſcend thro' a Quadrant of a Circle, whoſe Semidiameter is equal to the Length of the ſaid Pendulum; and, from *Simpſon*'s Fluxions, P. 538, it may be inferred, that if a denotes the Length of a ſimple Pendulum, and p the Semiperiphery of a Circle whoſe Radius is Unity, half the Time of an entire lateral Oſcillation will be defined by this Series $p \times \sqrt{\frac{1}{2}}\, a$ into $1 + \frac{1}{2.2.2} + \frac{3.3}{2.2.4.4.2^{2}} + \frac{3.3.5.5}{2.2.4.4.6.6.2^{3}} + \frac{3.3.5.5.7.7.}{2.2.4.4.6.6.8.8.2^{4}} + \&c.$ Wherefore ſince the Time in which a Body might freely deſcend thro' any given Right-Line $(2a)$ is known to be as $2\sqrt{2a}$, this therefore will be to the above-mentioned Time as $\frac{4}{p}$ to $1 + \frac{1}{2.2.2} + \frac{3.3}{2.2.4.4.2^{2}} + \frac{3.3.5.5}{2.2.4.4.6.6.2^{3}} + \&c.$

From

From whence, as the Time of falling thro' the Space $2a$ is really given, the exact Time of an Oscillation will likewise be had; that is, the Time in which the Beam would acquire a Vertical Position will become known.

The same answered by Mr. R———son.

Suppose that the Weights move in the Periphery of the Circle described by the End of the Beam in its Descent, and put $t = \sqrt{\frac{3}{16.5}}$ Seconds, the Time in which the Weights would descend 3 Feet perpendicularly by the uniform Force of Gravity: Then the Force whereby the Weight or Beam is accelerated in its Descent is to the Force of Gravity as $\frac{1}{3}$ to 1, and consequently the Distance the Weight would descend perpendicularly in the Time t by this Force continually acting upon it will be $\frac{1}{3}$ Feet: Whence $t\sqrt{3}$ will express the Time of descending 3 Feet perpendicularly by the constant and uniform Action of this Force; and therefore, by *Ex.* 2. Art. 355 of *Simpson*'s Fluxions, 108 : 100 : : $t\sqrt{3}$: $\frac{100t\sqrt{3}}{108}$ = the Time in which the Beam will acquire a Vertical Direction very nearly.

The same answered by ΦΙΛΟΠΟΝΟΣ.

Since the Times of Descent are as the Square Roots of the Distances perpendicularly descended, it will be as $\sqrt{39.2} : \sqrt{18} : : 1 : .6776$ the Time of Vibration of a Pendulum, whose Length is 18 Inches in an exceeding small Arc; but the Time of Vibration of any Pendulum in a Semicircle will be to the Time of Vibration in an exceeding small Arc as $1 + \frac{1}{4.2} + \frac{9}{4.16.4} + \frac{9 \cdot 25}{4.16.36.8} + \frac{9 \cdot 25 \cdot 49}{4\ 16.36.64.16}$

+

$+\frac{9.\ 25.49.\ 81.}{4.16.36.64.100.32}$ + &c. to 1, or 1.180367 to 1. Whence it is evident that the Time of Vibration of a Pendulum, whoſe Length is 18 Inches, in a Semicircle will be .799816.

Now the accelerative Force, by which the greater Weight deſcends, is to its abſolute Force as (6) the Difference of the Weights is to (30) their Sum, or as 1 to 5; therefore the Times of Deſcent (in all ſimilar Figures) being inverſely as the Square Roots of the Forces, we have $\sqrt{1} : \sqrt{5}$ :: .399908 (Half the Time laſt found) : .8942 the Time required.

PROBLEM XXXIII. *Anſwered by Mr. L. N—n.*

Put x for the Cones Altitude and y for the Semi-Diameter of the Baſe; then, if $\frac{.7854}{3}$ be denoted by p we ſhall have $py^2x = S$ and $\frac{4px^3+S}{5px^2}$ for the Diſtance of the Center of Oſcillation from the Vertex; which being a Minimum, its Fluxion $20px^4\dot{x} - 10\,Spx\dot{x}$ muſt be $= 0$, whence $x = \overline{\frac{S}{2p}}\Big|^{\frac{1}{3}}$.

R. R. H. anſwered this Problem after the very ſame Manner. $p = \frac{.7854}{3} = .2618$,

Put x, $ab = y$, $py^2x = S \therefore y^2 = \frac{S}{px}$.

Then $\frac{4x}{5} + \frac{y^2}{5x}$ a min.

Or $Ax + \frac{y^2}{x}$ a min. $= Ax + \frac{S}{px^2}$.

Influxions $A\dot{x} - \frac{2S\dot{x}}{px^3} = 0$, The

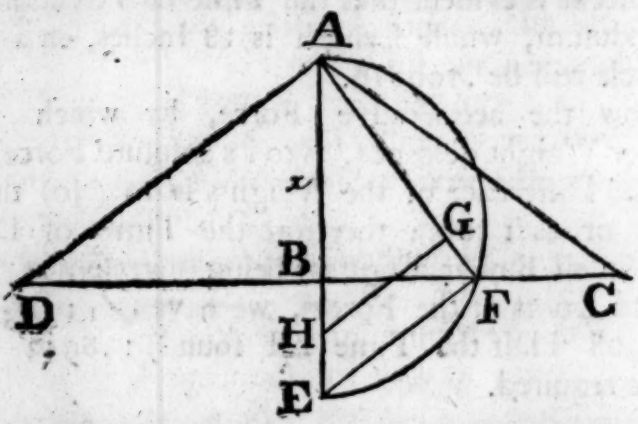
$2px^3 = S$, $\therefore x = \overline{\frac{S}{2p}}\Big|^{\frac{1}{3}}$.

$\therefore y^2 = \overline{\frac{2S^2}{p^2}}\Big|^{\frac{1}{3}}$, $\therefore py^2x = S$.

Then $\frac{4x}{5} + \frac{y^2}{5x} = \frac{4}{5}\cdot\overline{\frac{S}{2p}}\Big|^{\frac{1}{3}} + \frac{\overline{\frac{2S^2}{p^2}}\Big|^{\frac{1}{3}}}{5\cdot\overline{\frac{S}{2p}}\Big|^{\frac{1}{3}}} = \frac{4}{5}\cdot\overline{\frac{S}{2p}}\Big|^{\frac{1}{3}} + \overline{\frac{4S}{5p}}\Big|^{\frac{1}{3}} =$

$\overline{\frac{64S}{250p}}\Big|^{\frac{1}{3}} + \overline{\frac{8S}{250p}}\Big|^{\frac{1}{3}} = \overline{\frac{64}{65\cdot 45}}\Big|^{\frac{1}{3}} + \overline{\frac{45}{65\cdot 45}}\Big|^{\frac{1}{3}}$.

The same answered by Mr. Graham.



Let ADC be a Cone of the given Solidity and which being suspended at its Vertex will oscillate faster than any other Cone of the same Magnitude: Put AB the Axis of the Cone $= x$, and let p denote the Periphery of the Circle whose Radius is Unity; then will the Semidiameter of the Base $= \sqrt{\frac{3S}{px}}$. But the Distance of the Center of Oscillation from the Point of Suspension is known to be $\frac{4AB^2+BC^2}{5AB} = \frac{4x+\frac{3Sx^{-2}}{p}}{5}$; which must be a Minimum, since the Time of Oscillation is such; and therefore the Fluxion thereof being put $= 0$, after proper Reduction we have $x = \sqrt[3]{\frac{3S}{2p}}$; from whence and the given Solidity the Cone will be determined.

$p = 3.1416$, $py^2x = 3S \therefore y^2 = \frac{3S}{px}$, $y = \sqrt{\frac{3S}{px}} = \frac{1}{2}\sqrt{\frac{3S}{p}\cdot\sqrt[3]{\frac{3S}{2p}}}$

The same answered by ΦΙΛΟΞΕΝΟΣ.

In the indefinite Right Line AE, take BA to BE as 2 to 1, and on AE as a Diameter let a Semicircle be described; draw the Semi ordinate BF which produce, each Way, so that FC $=$ BF and

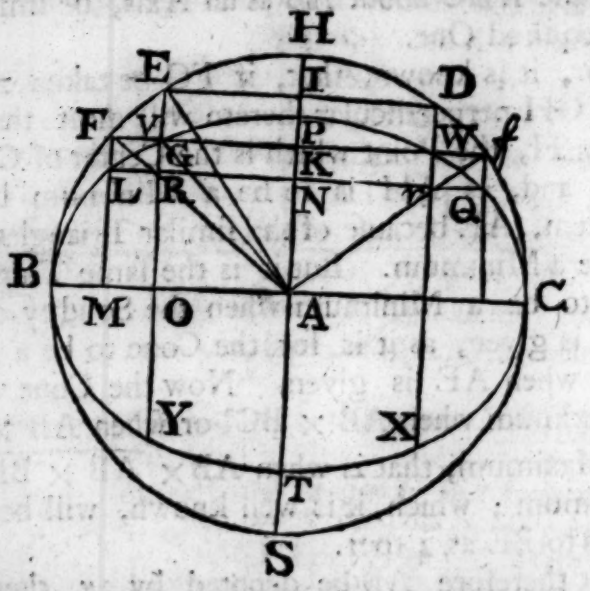
and BD = BC, and join D, A and A, C; then will the Cone, deſcribed by the Revolution of the Triangle ADC about AB as an Axis, be ſimilar to the required One.

For, it is known, that, if FG be taken = $\frac{1}{7}$ of AF, GH perpendicular thereto will meet the Axis AB in H, the Point which is the Center of Oſcillation; and, as AH is to be a Minimum by the Problem, AE becauſe of the ſimilar Triangles muſt alſo be a Minimum. But it is the ſame Thing for AE to be a Minimum when the Solidity of the Cone is given, as it is for the Cone to be a Maximum when AE is given. Now the Cone will be a Maximum when $AB \times BC^2$ or when $AB \times BF^2$ is a Maximum, that is when $AB \times \overline{AB \times BE}$ is a Maximum; which, it is well known, will be when AB is to BE as 2 to 1.

Let therefore AB be denoted by $2x$, then will $BC = 2x\sqrt{2}$ and the Circle deſcribed by $BC = 8px^2$; which multiplied by $\frac{1}{3}$ of AB gives $\frac{16px^3}{3}$ for the Content of the Solid. Hence $\frac{16px^3}{3} = S$ and $2x = \left.\frac{3S}{2p}\right|^{\frac{1}{3}}$.

Meſſ. *R*——*ſon* and *Leigh*, alſo, anſwered this Problem.

PROBLEM XXXIV. *Answered by* ΣΟΥΟΣ.



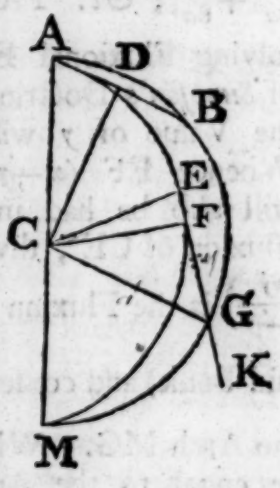
Let BPCT be the Spheroid, whereof the proposed Bottle VWXY is a Part; and let BHCS be a Circle described on the Transverse Axis thereof; then, if the Line LNQ limit the Surface of the Liquor: we shall have by the Problem 6 : 5 : : (PA : AN = ML) : : AH : FM; whence (if HA = 1) EO will be had = $\frac{1}{6}$ = .8333, EI = .55277, FM = $\frac{1}{5}$ = .6, NL = $\frac{4}{5}$ = .8, EG = .23333, FG = .24722, HK = .4, and BO = .44722; also HF = 53° 7′ 48″ and BE = 56° 26′ 33″. Now the Content of the Sphere, whose Radius is Unity, being 4,1888, the Content of the Segment thereof whose Versed Sine is BO (=.44722) will be equal to .5346, and that of the Segment whose Versed Sine is HK (.4) = .4355; therefore we shall have 4.1888 $\overline{PA}|^2$ for the Content of the whole Spheroid, .5346 $\overline{PA}|^2$ for the Content of the Segment VBYO thereof, and .4355 $\overline{PA}|^2$ for the Content of the Segment LPQN.

But

But by Problem 16, the Content of the Second Segment EGFG of the Sphere will be .0256, therefore that of the correſponding Segment of the Spheroid will be .0256 $\overline{PA}|^2$.

Moreover, by taking the Double of the Segment VBYO from the whole Spheroid, and the Double of the Segment VRLRV from the Segment LPQN, we have 3.1196 $\overline{PA}|^2$ for the Content of the Bottle and 3842 $\overline{PA}|^2$ for the Content of the Part RVPW*r*R drunk at the firſt Draught. Hence .3842 $\overline{PA}|^2$: 3.1196 $\overline{PA}|^2$ :: 1 : 8.12 the Number of Draughts required.

PROBLEM XXXV. *Aſwered by* ΦΙΛΟΠΟΝΟΣ.



Let CAGM be the propoſed Circle, and AEM the required Curve; alſo let E and G be any cotemporary Poſitions of the two Bodies, and from the Center C of the given Circle ſuppoſe CG, CE and CF to be drawn, the laſt of them perpendicular to EG. Put CG = *a*, GE = *x*, GF = *y*, and let the Velocity of the purſuing Body be denoted by *m*, and That of the purſued, by *n*. Then the relative Celerity of the Latter in the Direction EGK, with which It moves from the former Body at E, will be expreſſed by $\frac{CF}{CG} \times n$, or its Equal $\frac{n\sqrt{aa-yy}}{a}$; which, taken from the Velocity *m* of the purſuing Body, leaves $m - \frac{n\sqrt{aa-yy}}{a}$, for the true Celerity with which this laſt Body gains upon the Other, or that wherewith the Diſtance EG decreaſes.

creaſes. But CE : EF :: m : $\frac{m \times EF}{CE} = \frac{m \times \overline{x-y}}{\sqrt{a^2+x^2-2xy}}$ = the Velocity with which CE decreaſes.

Conſequently the Fluxion of EG is to That of CE, as $m - \frac{n\sqrt{aa-yy}}{a}$ to $\frac{mx-my}{\sqrt{aa+xx-2xy}}$; that is, $\dot{x} : \frac{x\dot{x}-x\dot{y}-y\dot{x}}{\sqrt{aa+xx-2xy}} :: m - \frac{n\sqrt{aa-yy}}{a} : \frac{mx-my}{\sqrt{aa+xx-2xy}}$. Whence, we get the following Equation $y\dot{x} + x\dot{y} - x\dot{x} = \frac{maxy}{n\sqrt{aa-yy}}$; which, by putting $\frac{m}{n} = r$, and converting the Radical Quantity into an Infinite Series, becomes $y\dot{x} + x\dot{y} - x\dot{x} = rx\dot{y}$ into $1 + \frac{y^2}{2a^2} + \frac{3y^4}{8a^4}$, &c. From which, by the Method of reſolving Fluxional Equations (ſee Sect. 2. Part 2. of *Simpſon*'s Doctrine and Application of Fluxions) the Value of y will be found in Terms of x. Whence EF $(x-y)$ and CE^2 $(a^2 + x^2 - 2xy)$ will alſo be had in Terms of x. But Half the Fluxion of CE^2, divided by EF (or its Equal $\dot{x} - \frac{x\dot{y}}{x-y}$) is the Fluxion of ME (ſee Art. 136 of the ſaid Book) and conſequently $\frac{\dot{x}}{r} - \frac{x\dot{y}}{r \times x-y}$ = That of the Arch MG. Whoſe Fluent will conſequently be equal to the Arch itſelf. By Means whereof ME will likewiſe be given.

But in Caſes where the Poſition of both the Bodies is given when the Chace begins, it may ſometimes be neceſſary (when the Series does not converge ſufficiently faſt) to find the Values of FG and the deſcribed Arch GB, correſponding to any given Value of EG, at two, or more, Operations.

Thus, ſuppoſing b, c, and d to denote any three correſponding Values of EC, FG and the Arch

Arch BG, let $b-z$, $c+v$, and $d+w$ be three other cotemporary Values of the ſaid Quantities reſpectively.

Then by ſubſtituting $b-z$, and $c+v$, for x and y reſpectively, in the Fluxional Equation given above, we have $\overline{b-c} \times \dot{z} + b\dot{v} - z\dot{z} - v\dot{z} - z\dot{v} = \frac{ra\dot{v} \times \overline{b-z}}{\sqrt{a^2-c^2-2cv-vv}} = \frac{rab\dot{v}}{f} - \frac{raz\dot{v}}{f} + \frac{rabcv\dot{v}}{f^3}$ &c. (by putting $f = \sqrt{a^2-c^2}$, and converting the Radical Quantity into a Series.) Whence by Reduction, and putting $b-c=g$, $\frac{ra}{f} - 1 = b$, and $\frac{rabc}{f^3} = k$, there ariſes $g\dot{z} - bb\dot{v} - z\dot{z} - v\dot{z} + bz\dot{v} - kv\dot{v}$, &c. $= 0$.

From this Equation v will be found in Terms of z; which Value being ſuppoſed ſmall, two Terms of the reſulting Series will be ſufficient. And then, by ſubſtituting theſe new Values of $b-z$, and $c+v$, inſtead of x and y in $\dot{x} - \frac{x\dot{y}}{x-y} = \dot{w}$, and taking the Fluent, the correſponding Value of w will alſo become known. And ſo we ſhall have three new Values of EG, FG, and the Arch BG; with which the Operation may be repeated at pleaſure, and other correſponding Values of thoſe Quantities may be found, by the very ſame Equation above derived.

As to the particular Caſe hinted at in the Problem (See the *Ladies Diary* 1748) the firſt Value of EG being given equal to (AB) the Chord of 30°; and That of FG equal to the Half of it (BD,) the Velocity of the purſuing Body to that of the Body purſued will be found to be in the Ratio of $\frac{CD}{AC} + \frac{AB}{BM}$ to Unity, very near; that is in Numbers, as 1.16 to 1.

Note,

Note, When the Velocity of the Body purſued is the greateſt of the Two, the required Curve will be a Spiral converging continually, nearer and nearer, to the Circumference of a Circle concentric with the given One; whoſe Radius is to That of the given One in the Ratio of the leſſer Velocity to the Greater.

ANSWERS

Answers to the Paradoxes proposed in the Second Number.

Paradox I. *Answered by Mr.* Widd.

ANY two Places, on the Northern Hemisphere, having 180 Degrees Difference of Longitude, bear directly North from each other.

The same answered by the Proposer Mr. Graham.

Any two Places in the Northern Hemisphere, under the same Meridian, but on different Sides of the Pole, bear directly North from each other.

Paradox II. *Answered by the Proposer and Mr.* Widd.

Any two Places on the Surface of our Globe, that are Diametrically opposite to each other, will have the required bearing.

The Enigmas were answered by the Proposer, who says that the First is a *Bird Cage*, and the Second a *Pillow*.

A Collection of Problems to be answered in the next Number.

Problem XXXVI. *By Mr.* Marshall *of* Blenchland.

A Gentleman left the Bulk of his Fortune, consisting of Money and Plate, in equal Shares among his three Sons: To the First he gave $\frac{1}{7}$ of his Money, $\frac{1}{3}$ of his Plate and 280 Ounces over; to the Second he gave $\frac{1}{4}$ of his Money, $\frac{1}{4}$ of his Plate and 480 Ounces over; and to the Third $\frac{1}{5}$ of his Money, $\frac{1}{5}$ of his Plate and 132 Ounces over. 'Tis required to determine what Quantity of Money and Plate the Gentleman left? supposing the Plate to be valued at Five Shillings the Ounce.

Problem XXXVII. *By Mr.* L. N——n *of* Camberwell.

The Difference of the Legs of a right-angled Triangle, and the Difference between Half the Hypothenuse and the Perpendicular, falling thereon, being given; to find the Triangle.

Problem XXXVIII. *By* Honestus.

To determine the Sides of a right-angled Triangle, in whole Numbers; whereof the Ratio of the two Legs shall, at least, be 10 times nearer the Ratio of 7 to 11, than is the Ratio of 28 to 45.

Problem XXXIX. *By Mr.* Moss.

If, from the Angular Points of any acute angled plane Triangle, Perpendiculars be drawn to the opposite

oppoſite Sides, they will interſect each other, in a Point, within the Triangle: Quere the Demonſtration?

Problem XL. *By Mr.* Moſs.

Admit that one Port bears from another W. S. W. and their Diſtance aſunder to be 9 Leagues; and that two Ships ſail from thence at the ſame Time, directly North: That from the northermoſt Port at the Rate of three Miles an Hour, and the other at the Rate of five Miles an Hour: 'Tis propoſed to find the Diſtance ſailed by each Ship, when They are both, at the ſame time, equally diſtant from an Iſland that bears N. W. of the ſouthermoſt Port and diſtant therefrom ſeven Leagues?

Problem XLI. By ΣΟΦΟΣ.

Three Points being given in Poſition; to draw a Line, through one of them, ſo that the Rectangle under the Perpendiculars, falling thereon from the other two Points, may be equal to a given Square.

Problem XLII. *By* Bidefordienſis.

Given $\sqrt[y]{x} + y^{\circ} = 2.8385$, and $\sqrt{x} + \sqrt[3]{x} = 7.3415$: To determine x and y.

Problem XLIII. *By* Waltonienſis.

If, from each Angular Point of any Triangle, a Line be drawn thro' any Point, within the Triangle, to terminate in the oppoſite Side, the Solid under the Parts thereof, intercepted betwixt the Angles and the ſaid Point, will be to the Solid under the three remaining Parts, intercepted betwixt the Sides

and the ſaid Point, as the Solid under the three Sides of the Triangle is to either of the (equal) Solids under the alternate Segments of the Sides. Required the Demonſtration?

Problem XLIV. *By* ΦΙΛΟΣ.

To find the greateſt Error of an Horizontal Dial made for the Latitude of 51° 32′; but placed in the Latitude of 54°?

Problem XLV. *By* Waltonienſis.

To one End of a Cord let a Ball be faſtned, and to the other a ſmooth flat Weight; let this Weight be laid upon the upper End of an inclined Plane, perfectly poliſhed. Then, if the Ratio of the Weight to the Ball be greater than that of Radius to the Sine of the Planes Inclination, the Weight will deſcend along the Plane and raiſe the Ball: Now ſuppoſe, the Inſtant the Ball is thus made to aſcend, it is alſo put into a Vibratory Motion; it is required to find how many Oſcillations it will make in its whole Aſcent; the Inclination of the Plane and the Length of the Cord being given?

Problem XLVI. *By Mr.* Graham.

The Ages of any propoſed Number of equal Lives being given; to find a general Theorem expreſſing the Number of Years to the End of which They have an equal Chance of living together? The Decrements of Life being ſuppoſed in Arithmetical Progreſſion.

Problem XLVII. *By Mr.* R. R. H. *of* Southwark.

A Privateer ſails from a certain Port, ſteering due Eaſt ſix Miles an Hour, by the Log, in a Current ſetting South four Miles an Hour: A Ship of

of War, two Hours afterwards, ſails from the ſame Port, in purſuit of the Privateer, and keeps bearing upon her, running at the Rate of eight Miles an Hour. In what Time will the Man of War come up with her Chace? And what the Nature of the Curve ſhe deſcribes?

Problem XLVIII. *By* ΦΙΛΟΞΕΝΟΣ.

Let z be the Arch of a Circle, whoſe Radius is r, and let there be a Curve of ſuch a Nature, that, its Abſciſſa being the ſame with That of the Arch z, its correſponding Ordinate may be every where $\frac{z^n}{r^{n-1}}$; 'tis propoſed to find the Area of this Curve?

Problem XLIX. *By* ΣΕΜΝΟΣ.

Required a general Theorem for finding the Convex Superficies of any upright Elliptical Cone?

Problem L. *By* Waltonienſis.

Angria the Pirate eſpying an *Eaſt-India* Veſſel, diſtant two Leagues to the South, running before a briſk Weſtern Gale, immediately purſued and continued bearing directly towards it; but finding it conſtantly kept the ſame Diſtance ahead of him, after four Hours, he gave over the Chace; in which Time the *Indiaman*, keeping a direct Courſe to the Eaſt, had run ten Leagues: Quere the Diſtance ſailed by the Pyrate?

Problem LI. *By* Lupus.

To find the Fluent of $\frac{a^6\dot{x}}{a^6+a^4x^2+a^2x^4+x^6}$, by Means of Circular Arcs and Logarithms.

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PROBLEM LII. *By* ΦΙΛΟΠΟΝΟΣ.

Suppoſing that a ſolid Sphere (of Ice, or other Matter) of the ſame Magnitude and common Denſity with the Earth, revolving about its Axis at the Rate of one Revolution in three Hours, is reduced to a State of Fluidity; to determine the Nature and Dimenſions of the Figure, which it will aſſume or converge to, and alſo the Time of Revolution when the Particles of the Fluid have acquired an Equilibrium, or may be conſidered as quieſcent with Reſpect to each other.

Some

Some REMARKS, *on a late ſcurrilous Pamphlet in Anſwer to the Obſervations publiſhed in our laſt Number in Defence of Mr.* Simpſon's *Fluxions*; *by* John Turner.

IN publiſhing the Obſervations abovementioned, I had no Deſign to engage in a Controverſy with a Perſon whoſe Character is ſo well known. My View therein was to ſhew to Thoſe, who are not ſufficient Judges of the Subject, how groſsly Mr. *Simpſon*'s Book of Fluxions had been miſrepreſented by the *imaginary* Cantabrigienſis, or the *real* Compiler of the Ladies Diary. But as I now find myſelf perſonally abuſed and treated in a very ſcurrilous Manner, I ſhall treſpaſs ſo far on my Readers, as to take up a few Pages here; to expoſe the *groſs* Stupidity and *ſhameful* Prevarication of an indefatigable and *unabaſhed* Antagoniſt.

To put the Matter, then, in the cleareſt Light poſſible, it will be proper to begin with Mr. *Simpſon*'s Definition of Fluxions, as the greateſt Point in Debate; which runs thus, "The Magnitude by which any flowing Quantity *would be* uniformly increaſed in a given Portion of Time, with the generating Celerity at any propoſed Poſition, or Inſtant (was it from thence to continue invariable) is the Fluxion of the ſaid Quantity at that Poſition, or Inſtant."

Before we proceed to examine the Objections brought againſt this Definition, let us ſee what Sir *Iſaac Newton* ſays on the Subject; whoſe Name and Authority the *pretended* Remarker makes an extraordinary Uſe of.

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In the ſecond Lemma of the ſecond Book of his Principia in deſcribing a new Kind of Calculation, he had diſcovered, he expreſſeth himſelf thus — *It will be the ſame Thing, if, inſtead of Moments* (uſed by himſelf,) *we uſe either the Velocities of the Increments and Decrements* (*which may alſo be called the Motions, Mutations, and Fluxions of Quantities*) *or any finite Quantities proportional to thoſe Velocities.*

Now it muſt appear to Every One that, what Mr. *Simpſon* defines (above) as Fluxions, are exactly ſuch *finite Quantities, proportional to the Velocities* as Sir *Iſaac Newton* here ſpeaks of; ſince it is well known that the Quantities produced, or the Diſtances deſcribed, in any given Time, by Motions uniformly continued, are, accurately, as the Velocities of the ſaid Motions.

It is clear from hence, that Mr. *Simpſon* has been very far from claſhing with Sir *Iſaac Newton*, in preferring the latter of the two Methods, pointed out by him, to the Former.— The only colourable Objection that has been produced againſt the Definition in Queſtion, is, that the Term *Fluxion* is there uſed in a different Senſe from That put upon it by moſt other Authors.

But as the Words *Velocity* and *Fluxion* are far from being exactly ſynonymous in the common Acceptation (as all, who know their Derivation, muſt allow) I can ſee no Reaſon why the Word *Fluxion*, as a Term of Art, may not as properly be uſed to ſignify the Quantity of a Mutation, as the Velocity or Degree of Swiftneſs whereby that Mutation is produced. As in Surgery is *actually* the Caſe. But let an Author's Terms be ever ſo ill choſen, yet all Mankind will allow that, if he defines thoſe Terms, he ought to be underſtood in his own Senſe of Them — Let us now ſee, in the

firſt

firſt Place, how *conſcientiouſly* our *candid* Remarker has acted in this Particular.

And here it will be proper to begin with his grand Objection, ſo often repeated, *that Mr.* Simpſon *has miſtaken the Effect for the Cauſe in his Definition of a Fluxion.* To lay open the Critic's Deſigns in this Objection, we muſt firſt endeavour to explain his Meaning a little, as to *Cauſe* and *Effect.*

By the *Effect*, then, we are to underſtand a certain uniformly-generated Space, or Increment (which Mr. *Simpſon*, above, defines by the Name of Fluxion) and by the Cauſe, the Velocity generating that Increment. Now, if Mr. *Simpſon* has really *miſtaken the Effect for the Cauſe*, it follows that he muſt have taken the Increment for the Velocity: Which, it is certain, he has no where done —— You will perhaps aſk, what the Objector drives at? Why, the Gentleman, becauſe the Word *Fluxion* (as is obſerved above) is uſed in a different Senſe by other Authors, as ſynonymous with Velocity, would very *modeſtly*, by impoſing their Meaning upon it, make Mr. *Simpſon* appear guilty of the moſt glaring Abſurdities. And we ſhall ſoon ſee what pretty Conſequences he draws, for the Author, by a *proper* Uſe of theſe two Words only.

The Compiler (ſays he in p. 11.) *has repreſented relative Swiftneſs* (here relative Swiftneſs is ſubſtituted for a Fluxion contrary to the expreſs Meaning of the Author) *at a Point of Space already paſſed over by a ſuppoſed Increaſe of Space, afterwards to be moved over by that very Swiftneſs* (which is here properly uſed as a Velocity.)

Again, at p. 17, he goes on. *The Author has given us a* falſe Idea *of the Nature of Fluxions, or has confounded the* Idea; *firſt by conſidering a Fluxion or Velocity* (again Fluxion or Velocity) *as mere'y abſolute, and not comparative; and next by*

defining that Fluxion, or Degree of Velocity (ſtill Fluxion or Velocity) *to be the Increment of a Magnitude.*

Egregious Prevarication! Why are two Words thus, eternally, coupled together as ſynonymous Terms, when the Author himſelf uſes them every where in a diſtinct Senſe? If, as all Mankind allow, nothing betrays a baſer Diſpoſition than endeavouring to hurt another by a wrong Interpretation of Words and Actions, what are we to think of a Perſon, who thus, by the moſt ſhameful and barefaced Prevarication, attempts to make an Author of Genius and Reputation, appear guilty of the moſt enormous Abſurdities?

Twenty other Places might be produced, where he ſtill goes on, in the ſame Strain: But, as I have already (I think) ſufficiently made good one Part of my Charge, relating to his Want of Candour and Veracity, I ſhall now proceed to the other; namely, his groſs Stupidity and want of Judgment. And here his Remarks on my Obſervations (printed in the laſt Number) furniſh ample Matter to work upon; eſpecially his Defence of his two *extraordinary* Poſitions: Whereof the former run thus; *That, in Quantities uniformly generated, the* Fluxion *muſt (according to the ſaid Definition) be the* Fluent itſelf, *or elſe a Part of it.*

To this I anſwered, *That the* Fluxion *is neither the* Fluent itſelf *nor a Part of it: It is a Quantity of the ſame Kind with the Fluent; but the Fluent being the Quantity* already *produced by the generating Point, Line or Surface, ſuppoſed* ſtill *in Motion, and the Fluxion* what *will ariſe, hereafter, from the Continuation of that Motion; the latter can no more be denominated a Part of the former than the enſuing Hour a Part of the Time part.*

He replies thus. John *ſays, the Fluent being the Quantity already produced* —— *Pray how was this*

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Quantity produced, by ſome magic *Art, without any Fluxion? I believe not.* Can any Thing be more ſenſeleſs, or groſsly ſtupid, than this Query? Have we not expreſsly ſaid above, that the Fluent, or Quantity ſpoken of, is produced by the generating Point, Line, or Surface?

But let us hear him further — *The Fluent* (continues he) *has in every Point of its Generation its proper Fluxion; and ſince we are told, That the Fluxion is the Magnitude by which the flowing Quantity* would be *uniformly increaſed in a given Time, it follows, that at the Expiration of that given Time, the flowing Quantity being always in Motion* actually is *uniformly increaſed by that Magnitude. And being thus actually increaſed through all theſe given Parts of Time, from the firſt to the laſt Point, all theſe Increaſes muſt make up the Fluent, or Thing generated; as the Sum of the Parts make up the Whole of any Thing.*

We grant that the Sum of the Parts of any Thing make up the Whole; but what has This to do in proving that a Quantity continually increaſing is made up of [redacted] Parts not yet ariſen, or exiſting, as he here, in a moſt ſophiſtical Manner, attempts to do: He ought, ſurely, to have known that, when we ſpeak of a Fluent, it is of the Quantity already produced, and not of the Whole compounded of it added to what will ariſe hereafter; for this would be conſidering it in the Light of a *conſtant* Quantity. But, ſuppoſe we were to grant him, *That the Fluxion actually is a Part of the Fluent*, what monſtrous Conſequences would follow from thence? Why, truly, none at all; unleſs the Author had confounded the *Part* with the *Whole*, as the *candid* Remarker would *modeſtly* inſinuate, by ſaying *That the Fluxion muſt be the Fluent itſelf or elſe a Part of it.*

Let us now follow him to his other Poſition, *viz.*

That, in other Quantities generated by a variable *Law, the Fluxion will not be a* real, *but an imaginary Thing.*

In my Anſwer to This, I put the following Query.

Does it follow, becauſe a Body, really, moves over a certain Diſtance, in a given Time, with an accelerated, or a retarded Velocity, that there is no Diſtance over which it might paſs in the ſame Time, with its firſt Velocity uniformly continued.

To which he replies, *Yes*; *but then*, (continues he) *it is an* imaginary *Diſtance in regard to the Motion of that Body.* What can he here mean *with regard to the Motion of the Body?* Unleſs it be, that the ſaid Diſtance is not *really* deſcribed by an uniform Motion of the Body; which is ſaying Nothing at all: The Point in Queſtion is not, whether the Deſcription be *real*, but whether The Diſtance *itſelf be ſo*: For, he aſſerts *That the Fluxion is no* real *but an imaginary Thing*: And the Fluxion (by the Author's Definition) is The Diſtance itſelf and not the Deſcription of it—Why did not the *deep ſighted* Objector go one Step further, and even deny the Exiſtence of the Fluent too? Which he might have done with equal Propriety; as the Generation of Geometrical Quantities by the Motion of Points, Lines and Surfaces, is but *imaginary*—I ſhould be exceeding glad to know how this *ſubtile* Caſuiſt would have us underſtand the common Terms, made uſe of to expreſs the Rate or Degree of Swiftneſs of a Thing—We ſay a Ball moves at the Rate of ſo many Feet in a Second—That a Ship ſails at the Rate of ſeven, eight or nine Knots (or Miles) an Hour; and ſo on. Are not we to look upon this Diſtance of ſeven, eight or nine Miles as the Space that *would be* deſcribed or run by the Ship, in the Time of one Hour, with the ſame Degree of Swiftneſs uniformly continued

An

And is this Rate, or Diſtance a Jot the leſs *real*, or harder to conceive, becauſe the Ship, by the Winds dying away, may afterwards ſlacken her Motion or, even, come to an Anchor before ſhe has run the Half of that Diſtance? Surely not. Fluxions, as Mr. *Simpſon* defines them, are theſe very Diſtances expreſſing the Rates of Motions in a given Time; and whoever diſputes the Reality or Exiſtence of the One (as this *wiſe* Antagoniſt has done) muſt alſo deny the Reality and Exiſtence of the Other. — So much with regard to his grand Poſitions; which he has endeavoured to ſupport by Equivocation and the *poor* Artifices of a little, ſhufling, Adverſary.

We now come to the Manner of correcting the Fluent objected to as ſpecified in our laſt Number: In reference to which he thus expreſſeth himſelf — *In* $y = -c \times \overline{xx + 2ax + 2aa} + d\dot{M}^{\frac{1}{2}}$ *the Compiler was told, that (though he propoſed to correct the Fluent by* $d\dot{M}^{\frac{1}{2}}$ *for* x *to be* $= o$ *when* $y = o$,) *there were infinite Caſes wherein* x *cannot be* $= o$, *when* $y = o$, *to which his Advocate, or himſelf only, anſwers* (ſtop a Moment, to ſee an Inſtance of the Critic's extraordinary Skill in juggling with Words and, even, with Points; For, though he places the *Comma* after the Word *only*, yet, it ſeems plain, he would have that Word underſtood as relative to *Anſwers* and not to the *Author himſelf*; in order to inſinuate an Untruth too notorious to be openly aſſerted without ſuch a Subterfuge——But he goes on) *that* x *and* y, *in moſt Caſes, are alike; and inſtead of candidly acknowledging his Miſtake, or falſe Step here made, as might have been expected from a F. R. S. Ribaldry and Equivocation are offered to ſupport Error.*

Now, if after all This, it ſhould be plainly made to appear that, there is neither a Miſtake nor falſe

falſe Step here made by the Author, nor Equivocation nor Ribaldry offered by myſelf in the Defence of him, what muſt the World think of the Judgment and Morals of a Man, who has even here (where he is *utterly* in the Dark) dared to deliver himſelf in ſo peremptory and arrogant a Manner? But, to come to the Point——Let b be be taken to expreſs the Value of y, when $x = 0$ *be that Value what it will:* Then, by ſubſtituting b for y, and 0 for x, the Equation will become $b = -c \times 2aa + d\,\dot{M}^{\frac{2}{2}} = -2ca^2 + d$; whence (to any One who knows what is meant by Tranſpoſition in Algebra) the Value of d will appear to be $= b + 2ca^2$; which Value, ſubſtituted for d, gives the Equation, $y = -c \times \overline{x^2 + 2ax + 2aa} + \overline{b + 2ca^2} \times \dot{M}^{\frac{2}{2}}$, truly and *univerſally* corrected, by Means of the Exponential, agreeable to what Mr. *Simpſon* has obſerved.

If my Antagoniſt can diſcover any Thing falſe or equivocal in This, I here promiſe, publickly, to acknowledge my Miſtake, and aſk his Pardon: But, in the mean Time, muſt take the Liberty to put him in Mind that, if, after all his extraordinary Confidence and Oſtentation, he is not able to point out the leaſt Flaw in it (which no Man living can) his Readers will have more Reaſon than ever to look upon him *as groſsly ignorant in the moſt common Parts of the Subject.*

There is alſo another Thing, if he has not given up all Pretenſions to Truth and Morality, that I be to remind him of; which is, that he would make it appear, as he ſays *it is known that Mr.* Simpſon *publiſhes Nothing but by the Aſſiſtance of Others*, who thoſe *Others* are to whom Mr. *Simpſon* is under ſuch *ſingular* Obligations —— I myſelf have had the Pleaſure of knowing Mr. *Simpſon*

ſon for ſome Years, and dare affirm in the Face of the World that, if ever Man alive was wronged by an Imputation of this Kind, Mr. *Simpſon* is.

If the *honeſt* Gentleman, who ſays *it is known that the Compiler* (meaning Mr. *Simpſon*) *publiſhes Nothing but by the Aſſiſtance of Others*, can make it appear who thoſe Aſſiſtants are, or ſhew that the Author's Copy has been reviſed or even ſo much as inſpected by any other Perſon whatever (except in Relation to his firſt Performance, where he was under a Neceſſity of ſubmitting to it) I ſhall make no Scruple to own that I have formed a very wrong Judgment of the Perſon whoſe Cauſe I have engaged in. On the other Hand, if he is not able to do it (as I am confident he is not) I leave it to be determined by Mankind whether the *Heart* or the *Head* of the Man is the worſer.

P. S. I muſt here take the Liberty to tell the Perſon, who at *p.* 34. of the *Palladium* for 1752. pretends to ſuch great Improvements in the Doctrine of Annuities, that he is not a Judge of the Subject, and that he merits the ſevereſt Contempt; firſt, for repreſenting Mr. *De-Moivre*'s Rule for the Value of a ſingle Life as *falſe*; and ſecondly, for expreſſing himſelf with an unjuſt and meanſpirited Bitterneſs againſt another Author of Reputation, from whom he never received the leaſt Provocation.

—— But what can be expected from the Principles of a Man, who makes no Scruple to propoſe the printing of *Mr.* Abraham De-Moivre's *Doctrine of Annuities, corrected and improved by himſelf:* Notwithſtanding the Author, Mr. *De-Moivre*, is ſtill living, and has lately obliged the World with a new Edition of that very Work.

When

When I ſeriouſly reflect on the unbounded Licence that is (now) allowed, in the *Diary* and *Palladium*, for little Pretenders and nameleſs Wretches to be ſcurrilous and throw Dirt at Perſons of ſuperiour and approved Genius; I cannot help conſidering *thoſe* Performances, under the Hands of the preſent Conductor, in the Light of Publick Vaults, or Neceſſary Houſes that are open for every dirty F—ll—w to ſh—t in.

A LETTER *to the* AUTHOR.

By HONESTUS.

SIR,

AS you were pleaſed to allow a Place in your laſt Number to my Animadverſions on the Want of Judgment, and the inſufferable Inſolence of the preſent Compiler of the *Ladies Diary*; I hope you will now give me Leave to lay before the Publick ſome further Remarks on the Conduct and *ſingular* Productions of this extraordinary Perſon, in ſome late *Diaries*, under his Direction.

The firſt Thing I ſhall here take Notice of is Queſtion I. in the *Diary* for 1748 propoſed by himſelf, in Numbers abſolutely impoſſible: Notwithſtanding which he has been ſo civil (at *p.* 21. of the ſucceeding *Diary*) to *deſire all Thoſe who anſwered it* wrong *not to be diſobliged at their Solutions being omitted, as he is not againſt their* ſhining *in a proper Place.* His Modeſty here is very remarkable: Theſe ſame Perſons, to whom he is ſo complaiſant to allow the Liberty of *ſhining* in a proper Place, had, doubtleſs, been free enough to tell him of his Error; upon which, having firſt *corrected the Data* (a

new

new Term of Art introduced by himſelf) he there lets Them, and the World, know They are quite ignorant of the Matter; however to ſhew his good Nature, at the ſame Time, is not againſt *their* ſhining *in a proper Place.*

In the very ſame Page he is foul on one of his Contributors for giving (as he pretends) a falſe Solution; though what his Correſpondent advances is perfectly right, and what he himſelf remarks thereon abſolutely falſe and inconſiſtent ——Would any Body, but himſelf have aſſerted that the Number 89, divided by 6, 5, 4, 3 and 2, ſucceſſively, leaves 5, 4, 3, 2, and 1, reſpectively, for Remainders; and upon ſo ſlender and even falſe a Foundation have reflected on the Judgment of Another, who was perfectly right in his Solution.

The 12th Queſtion for the above mentioned Year (1748) is another of his own propoſing, and is *ſtill* worſe than the Firſt, being not only impoſſible with reſpect to particular Numbers but abſurd in the very Nature of it: Yet even here he has been ſo far from making thoſe aukward Apologies which ſome *over* modeſt Perſons would have made, that he riſes greatly conſpicuous by his own ignorant Blunders: For having firſt (as uſual) *corrected the Data* he ſolves it his own way, and then very modeſtly tells the World, it was

Anſwer'd by Mr. Heath *only.*

Such is the admirable Addreſs and Subtlety of the Man, in rendering what Others would bluſh, and be confounded at, ſubſervient to his invincible Deſire of appearing conſiderable, in Spite of Nature.

To be continued.

ERRATA, in N°. II.

IN Page 8, Line 22, for $65° 26'$ read $54° 53'$; l. 23, for $7^H 38^M 16^S$ r. $8^H 20^M 18^S$; l. 24, for $4^H 21^M 44^S$ r. $3^H 39^M 32^S$; l. 35, for $19° 14'$ r. $25° 46'$; l. laſt, for $65° 35'$ r. $55° 7'$; in l. 2. p. 9, for $4^H 22^M 2^S$ r. $3^H 40^M 28^S$; l. 5. p. 29, for 3 read 2; l. 3. p. 31, for *Obſervations* r. *Calculations*.

ERRATA in N°. III.

IN the 18th and laſt Lines of Page 18 of the Dialling, for *Reclining* r. *Inclining*; p. 19 of the Series, l. 18, for $\frac{1}{1-yn}$ r. $\frac{1}{1-y^n}$, l. 2, from Bottom, for $\overline{1-n}|^2$ r. $\overline{1-y^n}|^2$; p. 20. l. 11, for $y2^n$ r. y^{2n}; p. 21. l. 2, for $x2$, $x3$, $x4$, r. x^2, x^3, and x^4 reſpectively; p. 23. l. 8, for $\frac{yxn}{n}$ r. $\frac{yx^n}{n}$.

MATHEMATICAL EXERCISES:

CONTAINING,

I. The Principles of Dialling.

II. A Method to find a Vulgar Fraction nearly equal in Value to any Decimal One.

III. Some Obſervations on Fluents.

IV. Anſwers to the Problems propoſed in the third Number.

V. A Collection of new Problems.

VI. Memorial Verſes, adapted to the new Style.

By JOHN TURNER.

No. IV.

LONDON:

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[Price ONE SHILLING.]

ERRATA in N°. III.

PAGE 68, Line 8, for *faster* read *fastest*
25, for 60° r. 6°
72, l. 18 and 22, for $\sqrt{3}$ r. $\sqrt{5}$
73, l. 9, after *similar* r. *and equal*
77, l. 15, for F r. E
N°. 4. Page 107, l. 7, before *Difference* r. g
and l. 8; dele *given*.

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The fifth Number being to be publi
about the Beginning of *December* ensuing.

A

METHOD

To determine a

VULGAR FRACTION

Nearly equal in Value to any DECIMAL ONE *propoſed.*

LET $\frac{a}{b}$ and $\frac{c}{d}$ be aſſumed to denote two Vulgar Fractions, in the loweſt Numbers, equal nearly to the firſt Figures of the given Decimal; the one of them greater, and the other leſs, than the ſaid Decimal: Moreover, ſuppoſing $\frac{c}{d}$ to be the neareſt Value of the two, and the given Quantity (or Decimal) to be denoted by q; let there be aſſumed $\frac{a+cx}{b+dx} = q$: Whence x will be had $= \frac{bq-a}{c-dq} = \frac{a-bq}{dq-c}$. Let this Value of x be found in the neareſt whole Numbers; by Means whereof the Value of the Fraction $\frac{a+cx}{b+dx}$ (which is a firſt Approximation to the Value q) will be given. And, if with the Fraction thus determined, and

one of the two preceding ones $\left(\frac{a}{b}\text{ or }\frac{c}{d}\right)$ the Operation be repeated, a new Value of x will be had from the very Equation above exhibited; and from thence a ſecond Approximation or Value of the aſſumed Fraction $\frac{a+cx}{b+dx}$. Whence the Operation, (if neceſſary) may be again repeated, till you arrive to any Degree of Exactneſs required.

Example I.

Let the Quantity propounded be 3.14159 26; expreſſing the Periphery of a Circle whoſe Diameter is Unity.

Here it is plain that $\frac{a}{b}$ will be $\frac{4}{1}$, and $\frac{c}{d}=\frac{3}{1}$: Therefore, by writing 4, 1, 3, 1, and 3.14 &c. in the Room of a, b, c, d, and q reſpectively, the Value of x $\left(=\frac{a-bq}{dq-c}\right)$ is found $=\frac{4-3.13\ \&c.}{3.14\ \&c.-3}$ $=\frac{.85\ \&c.}{.14\ \&c.}=6$, nearly. Whence we have $\frac{a+cx}{b+dx}$ $=\frac{4+3\times 6}{1+3\times 2}=\frac{22}{7}$; for a firſt Approximation, being the ſame with that of *Archimedes*.

Again, by taking $\frac{a}{b}=\frac{3}{1}$, and $\frac{c}{d}=\frac{22}{7}$ and ſubſtituting as before, we get $x\left(=\frac{a-bq}{dq-c}\right)=\frac{3-3.14159}{7\times 3.14159-22}$ $=16$ nearly. Whence we have $\frac{a+cx}{b+dx}=\frac{3+352}{1+112}$ $=\frac{355}{113}$, for a ſecond Approximation; agreeing with That of *Metius*. Which, converted to a Decimal, errs only in the 7th Place.

Example

Example II.

Let the Quantity to be approximated be 3.43406.

Here $\frac{a}{b} = \frac{4}{1}$, and $\frac{c}{d} = \frac{3}{1}$; whence $x\left(\frac{a-bq}{dq-c}\right) = 1$; And ſo $\frac{a+cx}{b+dx} = \frac{7}{2}$, for a firſt Approximation.

Secondly $\frac{c}{d} = \frac{7}{2}$, and $\frac{a}{b} = \frac{3}{1}$: Therefore $x\left(\frac{a-bq}{dq-c}\right) = 3$; and $\frac{a+cx}{b+dx} = \frac{24}{7}$; for a ſecond Approximation.

Thirdly $\frac{c}{d} = \frac{24}{7}$, and $\frac{a}{b} = \frac{7}{2}$: And ſo $x = 3$, and $\frac{a+cx}{b+dx} = \frac{79}{23}$; a third Approximation.

Fourthly $\frac{c}{d} = \frac{79}{23}$, and $\frac{a}{b} = \frac{24}{7}$; whence $x = 2$, and $\frac{a+cx}{b+dx} = \frac{182}{53}$; a fourth Approximation.

Fifthly $\frac{c}{d} = \frac{182}{53}$, and $\frac{a}{b} = \frac{79}{23}$; whence $x = 3$, and $\frac{a+cx}{b+dx} = \frac{79+546}{23+159} = \frac{625}{182}$; for a fifth Approximation.

Some Obſervations concerning FLUENTS.

By Waltonienſis.

AN Equation of Fluents, deduced from a Fluxionary Equation, very often admits of various Corrections, according to the various Problems in whoſe Solutions ſuch Equation may ariſe: And, conſidering the Equation independently of the Problem it may be derived from, we may ſuppoſe one of the ſimple variable Quantities to have its Beginning when the Other has attained

any Value whatever. This the Writers on Fluxions have adverted to and explained. But a Fluxionary Equation will ſometimes admit only of one particular Equation of the Fluents; one of the ſimple variable Quantities neceſſarily having its Beginning with the Other, or when the Other has attained one certain Value. Which is a property I do not find thoſe Writers have taken any Notice of.

EXAMPLE I.

The Equation $\frac{a\dot{x}^2}{\dot{y}} = y\dot{x} - x\dot{y} + b\dot{y}$ being propoſed, to find the Equation of the Fluents; if $\dot{y}$ be ſuppoſed invariable, and the Fluxions of both Sides be taken, we have $\frac{2a\dot{x}\ddot{x}}{\dot{y}} = y\ddot{x} + \dot{y}\dot{x} - \dot{x}\dot{y} = y\ddot{x}$, and $2a\dot{x} = y\dot{y}$, of which Equation, it is obvious, the Fluents are $4ax = y^2 + c$, c being ſome conſtant Quantity. But here c cannot be any conſtant Quantity whatever, it can only have one certain Value, *viz.* $4ab$ (as appears by ſubſtituting in the original Equation) and the Equation of the Fluents is $4ax = y^2 + 4ab$, and no other; for y muſt neceſſarily begin when x is $= b$. Suppoſing $b = 0$, the Fluxionary Equation becomes $\frac{a\dot{x}^2}{\dot{y}} = y\dot{x} - x\dot{y}$; and the Equation of the Fluents $4ax = y^2$; which will not admit of the Addition or Subſtraction of any conſtant Quantity, for y muſt neceſſarily begin when x begins.

EXAMPLE II.

The Equation $\frac{a\dot{x} + y\dot{x}}{\dot{y}} + b = x + y - \frac{x\dot{y}}{\dot{x}}$ being propoſed, to find the Equation of the Fluents; ſup-

ſuppoſing $\dot{y}$ invariable, and taking the Fluxions of both Sides, we ſhall, by ordering them properly, get $\frac{\dot{x}}{x^{\frac{1}{2}}} = \frac{\dot{y}}{\overline{a+y}^{\frac{1}{2}}}$; of which it is eaſy to perceive the Fluents are $x^{\frac{1}{2}} = \overline{a+y}^{\frac{1}{2}} - c$, c being again put for ſome conſtant Quantity. But here, again, c cannot be any invariable Quantity whatſoever; it can only be $= \overline{a+b}^{\frac{1}{2}}$ (as appears by ſubſtituting properly in the original Equation) x of Neceſſity having its Beginning when y is $= b$, and the Equation of the Fluents can be no other than $x^{\frac{1}{2}} = \overline{a+y}^{\frac{1}{2}} - \overline{a+b}^{\frac{1}{2}}$. Suppoſing $b = 0$, the Fluxionary Equation becomes $\frac{a\dot{x}+y\dot{x}}{\dot{y}} = x+y-\frac{x\dot{y}}{\dot{x}}$ (which is That propoſed by Mr. *Emerſon*, page 50 of his Fluxions) and the Equation of the Fluents will be $x^{\frac{1}{2}} = \overline{a+y}^{\frac{1}{2}} - a^{\frac{1}{2}}$, and no other; for y muſt neceſſarily begin when x begins.——This Mr. *Emerſon* ſeems to have been ignorant of, and erroneouſly makes the Equation of the Fluents $x = a+y$ (an Equation to a Right-Line, inſtead of an Equation to a Parabola). The Falſity of which he might eaſily have perceived; for x being $= a+y$, $\dot{x}$ muſt be $= \dot{y}$, which Values of x and $\dot{x}$ being put, in their Stead, in the original Equation, it becomes $\overline{a+y} \times \frac{\dot{y}}{\dot{y}} = a+y+y-\overline{a+y} \times \frac{\dot{y}}{\dot{y}}$; according to which $2a+2y = a+2y$, and $a = 0$; which is abſurd, ſince it is plain a is put to denote ſome Quantity.

ANSWERS

ANSWERS *to the* PROBLEMS *propoſed in th* THIRD NUMBER.

PROBLEM XXXVI. *Anſwered by Mr.* G. Brown bridge *of* Ellerton.

LET the Value of 280, 480 and 132 Ounce of Plate, in Pounds Sterling, be denoted b *a*, *b* and *c* reſpectively; alſo let *x* repreſent th Value of all the Plate, and *y* the Quantity o Money. Then $\frac{y}{7} + \frac{x}{3} + a = \frac{y}{4} + \frac{x}{4}$ $+ b = \frac{y}{5} + \frac{x}{3} + c$, by the Problem; and there fore $\frac{y}{7} + a = \frac{y}{5} + c$; whence $y = \frac{35a - 35}{2}$ $= £ 647.5$. Now for $\frac{y}{4} - \frac{y}{5}$ $(= 32.375$ ſubſtitute *d*, and we ſhall have $d + \frac{x}{4} + b = \frac{x}{3}$ $+ c$; whence $x = 12d + 12b - 12c = £ 1432.5$ or 5730 Ounces, the Quantity of Silver ſought.

The ſame anſwered by Mr. Een. Bouw-man.

Let *x* repreſent the Value of the Money in Crowns, and *y* the Ounces of Plate; then, by th Problem, the firſt Son's Share, or $\frac{x}{7} + \frac{y}{3} +$ 280 will be = ſecond Son's Share, or $\frac{x}{4} + \frac{y}{4}$ $+ 480$, = third Son's Share, or $\frac{x}{5} + \frac{y}{3}$ $+ 132$

+132. Therefore, from the firſt and laſt, we have $\frac{x}{5} - \frac{x}{7} = 280 - 132 = 148$, whence $x = \frac{148 \times 35}{2} = 2590$ Crowns: Alſo from the ſecond nd third we have $\frac{x}{4} - \frac{x}{5} + 480 - 132 = \frac{y}{3} - \frac{y}{4}$, or $\frac{x}{20} + 348 = \frac{y}{12}$; whence $y = 27.5 \times 12 = 5730$ Ounces of Silver.

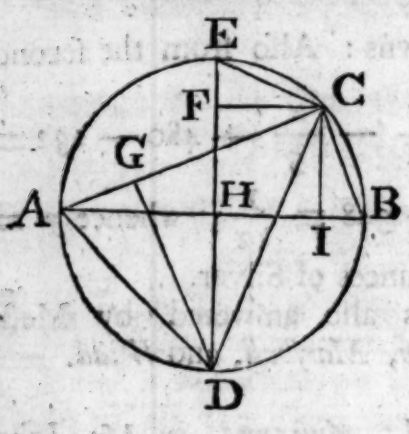
This Problem was alſo anſwered by Meſſ. *Noorthouck L. N——n*, *Marſhall*, and *Widd*.

PROBLEM XXXVII. *Anſwered by Mr.* John Noorthouck, *of* Watford.

Let x and $x+b$ repreſent the Legs of the Tri-ngle, y the Perpendicular and $y+c$ Half the Hypothenuſe; then $2x^2+2bx+b^2 =$ (by 47 *Eu.* 1.) $y^2+8cy+4c^2$; but $x^2+bx=2y^2+2yc$, or $2x^2+bx=4y^2+4cy$; therefore, if this laſt Equation e ſubſtracted from the firſt, we ſhall have $4cy+c^2=b^2$, and $y=\frac{b^2-4c^2}{4c}$; whence the Hypothe-nuſe and the Legs of the Triangle may be found.

The

The same answered by the Proposer, *Mr.* L. N—n.



Let AC = BC = a, BH—CI = n, AC = y, and BH = x; then will AB = $2x$, $y^2 - a^2 = 2x^2 - 2xn$, and $4x^2 = 2y^2 - 2a^2 + aa$: Whence taking the Double of the first Equation from the Latter, we have $4xn = a^2$, and therefore $x = \frac{a^2}{4n}$.

For the Solution of this Problem, Mr. *Leigh* gives the following

THEOREM.

As Half the Difference of the Sides, is to the Difference between Half the Hypothenuse and the Perpendicular, so is the Sine of (45°) Half the Sum of the Angles at the Base, to the Sine of Half their Difference.

The same answered by Proteus.

Let ABCD be a Circle described about the required Triangle ABC, whose Hypothenuse is AB and Perpendicular CI: Draw the Diameter EHD perpendicular to AB, and make HF = CI: Join C,E; C,D; C,F and D,A and draw DG perpendicular to AC; then AG will be equal to Half the Difference of the Sides, and EF to the Difference between Half the Hypothenuse and the Perpendicular. Since the Triangles AGD

DEC

DEC and EFC are ſimilar, we have AD : DE : : 1 : $\sqrt{2}$: : AG : EC. Whence the following

CONSTRUCTION.

Draw the indefinite Right-Lines DE, FC perpendicular to each other, and take FE equal to the Difference between Half the Hypothenuſe and the given Perpendicular: Upon E with the Radius EC, which is equal to a Fourth Proportional to 1, $\sqrt{2}$ and Half the Difference of the Sides, deſcribe an Arch interſecting FC in C: Make CD perpendicular to EC, and thro' E, C and D let the Circle ECBD be deſcribed; then, making the Diameter AB perpendicular to ED, join A, C and B, C and ABC will be the Triangle required.

Method of Calculation.

AD : DE : : 1 : $\sqrt{2}$: : AG : EC $=$ AG$\sqrt{2}$, and EC : EF : : Rad : Sine of ECF, Half the Difference of the Angles at the Baſe.

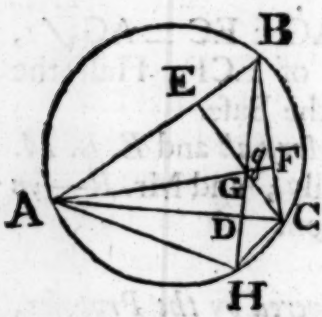
Meſſ. *Widd*, *Brownbridge*, *Marſhall* and *E. B. M.* ſolved this Problem algebraically; and Mr. *R—ſon* ſent a Geometrical Conſtruction.

PROBLEM XXXVIII. *Anſwered by the Propoſer*, Honeſtus.

Since the Square of n^2+m^2 is equal to the Sum of the Squares of n^2-m^2 and $2mn$, it is evident, that if the two Legs of the Triangle be expreſſed by n^2-m^2 and $2mn$, the Hypothenuſe will be expreſſed by n^2+m^2. Hence, by making $n^2-m^2 : 2mn$: : 11 : 7, we have $22mn = 7n^2 - 7m^2$ and $7n^2 - 22mn = 7m^2$, whence $n^2 - \frac{22}{7}mn + \frac{121m^2}{49} =$

$= \frac{121m^2}{49} + m^2$, and $n = \frac{m\sqrt{170}+11}{7}$; consequently $\frac{n}{m} = \frac{\sqrt{170}+11}{7} = 3.43406$ nearly. Whence it appears that $\frac{7}{2}$, $\frac{24}{7}$, $\frac{79}{23}$, $\frac{182}{53}$ and $\frac{625}{182}$ are so many approximated Values of the Decimal, by the second Example, p. 101. according to which the two Legs will appear to be 28 and 45, 336 and 527, 12719 and 19992, 19292 and 30315, 227500 and 357501; whereof the Ratio will be as 7 to $11\frac{1}{2}$, 7 to $10\frac{7}{24}$, 7 to $11\frac{5}{1817}$, 7 to $10\frac{3715}{17518}$, and 7 to $11\frac{1}{32500}$, respectively: Which last is 8000 times nearer the proposed Ratio of 7 to 11, than is 7 to $11\frac{1}{4}$, or 28 to 45.

Problem XXXIX. *Answered by* ΣΟΦΟΣ.



Let ABC be the Triangle, and let AF, BD and CE be Perpendiculars drawn from A, B and C respectively to the opposite Sides: Then, by similarity of Triangles AB : BD : : Bg : BE, and AB : BF : : BC : BE; whence AB × BE = BD × Bg = BF × BC, and consequently BD : BC : : BF : Bg. But BD : BC : : BF : BG, therefore Bg = BG, and G and g coincide. *Q. E. D.*

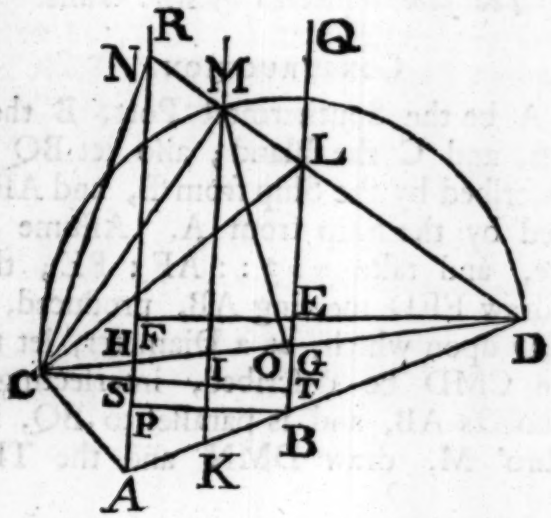
The same answered by Mr. R——son.

Suppose the Triangle ABC inscribed in a Circle, and drawing the Perpendiculars BD, CF, thro' G, their Point of Intersection, let AF be drawn:

drawn: Produce BD till it meets the Circumference in H, and let the Points A,H and C,H be joined. Then the Triangles BEG, DHC will be ſimilar, becauſe EBG=HCD (inſiſting on the ſame Arc AH) and the Angles at E and D are Right by Conſtruction. Whence the Angle CHD=EGB=CGH, conſequently the Triangle GAH will be an Iſoſceles One, and AHG=AGH=BGF: But the Angles GBF, DAH, ſtanding on the ſame Arc HC, are equal; therefore the Angle F=D, a Right-Angle by Conſtruction. *Q. E. D.* Mr. *L. N——n* and *R. R. H.* ſent Demonſtrations to this Problem.

PROBLEM XL. *Anſwered by Mr.* Widd.

In the Triangle ABP all the Angles and the Side AB are given, whence AP is found = 10.33 and BP = ST = 24.94: Alſo in the Triangle



ACS all the Angles and the Side AC are given, whence AS=CS is found=14.84; then CS+ST

 =CT

$=CT=39.79$. Now ſuppoſe LB to be denoted by x, AR will be $=\frac{5x}{3}$ and $LT=x-4.51$, alſo $RS=\frac{5x}{3}-14.841$: But (by the Problem) $CT^2+TL^2=CS^2+SR^2$; that is $x^2-9.03x+1603.96=\frac{25}{9}x^2-49.49x+440.99$; from which Equation the Value of x is eaſily found to be $=39\frac{1}{3}$ Miles, the Diſtance ſailed by the Ship from the Northermoſt Port; conſequently the Diſtance ſailed by the other Ship is $65\frac{1}{3}$ Miles.

Mr. *E. B. M.* ſolved this Problem Algebraically, and makes the Diſtance ſailed by the Ship from the Northermoſt Port $=39.37$ Miles, and that by the Ship from the Southermoſt Port $=65.62$ Miles: Mr. *Brownbridge* alſo ſent an Algebraic Solution.

The ſame anſwered by Mr. Ghiſs.

CONSTRUCTION.

Let A be the Southermoſt Port, B the Northermoſt, and C the Iſland; alſo let BQ be the Path deſcribed by the Ship from B, and AR That deſcribed by the Ship from A. Aſſume AF at Pleaſure, and take $5:3::AF:BE$; thro' F and E draw FED meeting AB, produced, in D: Join C,D upon which, as a Diameter, let the Semicircle CMD be deſcribed, interſecting KM, which biſects AB, and is parallel to BQ, in M; then thro' M, draw DMN and the Thing is done.

DEMONSTRATION.

From L, the Point where DN interſects BQ, draw LC; alſo draw CN and CM. Becauſe CMD

CMD is a Right-Angle, and ML=MN, it appears that CN=CL. Moreover, ſince AF : BE :: 5 : 3 by Conſtruction, it follows that FN : LE : : by reaſon of the ſimilar Triangles, AN : BL : : 5 : 3. *Q. E. D.*

Calculation.

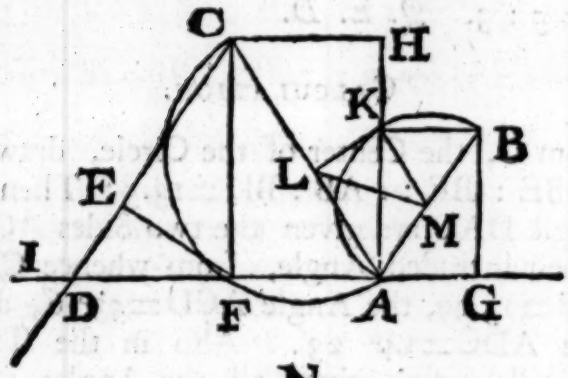
From O, the Center of the Circle, draw OM. AF—BE : BE : : AB : BD=13.5: Then in the Triangle DAC are given the two Sides AC, AD and their included Angle, from whence CD will be had=25.99, the Angle ACD=53° 6′, and the Angle ADC=14° 23′. Alſo in the Triangle GBD will be now given all the Angles and the Side BD, from which GD and ID will be found =12.59 and 16.79 reſpectively, and from thence IO=ID—OD=3.8. Now in the Triangle IMO there are given the Angle MIO, and the two Sides IO, OM, whence MOI will be had = 81° 17′, and from thence BL will be found =13.117, and AN=21.862, alſo CL=CN=17.587 Leagues.

Mr. *R——ſon* conſtructed this Problem in the ſame Manner as the above.

Problem

PROBLEM XLI. *Answered by Mr.* L. N—n.

CONSTRUCTION.



N

Join the given Points A,B and A,C with Right-Lines, and let A be the Point thro' which the required Line is to be drawn. Make CD parallel to BA, and take CE a Third-Proportional to AB and M the Side of the given Square. Upon AC describe a Semicircle, and from the Point E erect the Perpendicular EF, meeting the Semicircle in F; thro' A and F draw the Right-Line AI, and the Thing is done.

DEMONSTRATION.

Draw the Perpendiculars BG and CF. By Construction, AB : M : : M : CE, and by similar Triangles AB : BG : : CF : CE, therefore BG $\times$ CF = AB $\times$ CE = M^2. *Q. E. D.*

R. R. H. answered this Problem after the very same Manner.

The same answered by ΦΙΛΟΠΟΝΟΣ.

Let A,B and C be the given Points, FAG the required Line, CF and BG Perpendiculars, from the

he Points C,B to that Line, and N the Side of he given Square: Make AH perpendicular to FG, and equal to CF, and take AK equal to BG; alſo draw KL perpendicular to AC. Since $CF \times BG = N^2 = AH \times AK$, and, by ſimilar Tri-ngles, AC : AH : : AK : AL, it follows that $AC \times AL = AH \times AK = N^2$. Whence this

Construction.

In AC take AL a Third-Proportional to AC nd N, make LK perpendicular to AC, and on AB deſcribe a Semicircle interſecting KL in K; oin B,K and thro' A, parallel to BK, draw FG, nd the Thing is done.

Method of Calculation.

Draw LM to biſect AB in M and join M, K. n the Triangle LMA are given the Sides LA, AM with their included Angle, whence the Angles ALM, AML and conſequently KLM (=KLA -ALM) may be had. Then, becauſe MK= MA and LM common, it will be as AM : LM : *Sine*, ALM : *Sine*, LAM : : *Sine*, KLM : *Sine*, LKM, which being found LMK will likewiſe be found; as will alſo KMA and conſequently KAM (= the Complement of ½KMA.)

Mr. *R—ſon* alſo ſolved this Problem Geometrically.

Problem XLII. *Anſwered by the Propoſer.*

Let $z^6 = x$; then we ſhall have $z^3 + z^2 = 7.3415$, whence $z^6 = x = 21$; which ſubſtituted in the firſt Equation gives $\overline{21}^{\frac{1}{y}} = 1.8385$. Put l for the Logarithm of 21, Y for the Logarithm of y, and L for the Logarithm of 1.8385, then $Y = \frac{l}{L} = 5$.

The

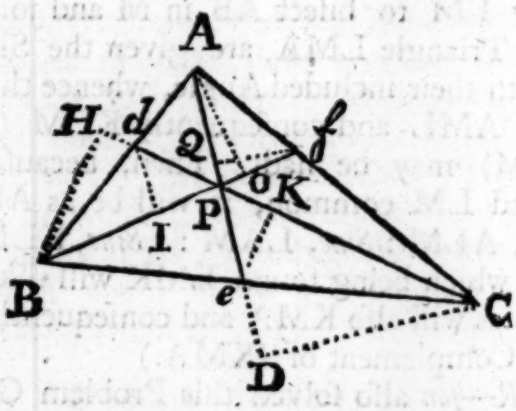
The same answered by Mr. Widd.

Suppose $z^2=\sqrt[3]{x}$; then $z^3+z^2=7.3415$, and $z=1.661$, whence $x=21$. Again $\sqrt[y]{x}+y^3=\sqrt[y]{x}+1=2.8385$; therefore $\sqrt[y]{x}=1.8385$ and $y=\frac{\text{Log. } x}{\text{Log. } 1.8385}=5$.

Mr. *L. N*——*n* also answered this Problem.

PROBLEM XLIII. *Answered by the Proposer,* Waltonienſis.

Conceive three ſuch Weights x, y, and z to be poſited at the Points A, B and C reſpectively,



that P may be their Center of Gravity: Then by Mechanics $x : y+z :: eP : AP$

$y : x+z :: fP : BP$

$z : x+y :: dP : CP$. Therefore

$x\times y\times z : \overline{y+z}\times\overline{x+z}\times\overline{x+y} :: Pe\times Pf\times Pd : AP\times BP\times CP$; and ſince

$x : y :: Bd : Ad$
$y : z :: Ce : Be$
$z : x :: Af : Cf$ } we have { $x : x+y :: Bd : AB$
$y : y+z :: Ce : BC$
$z : x+z :: Af : CA$

conſe-

conſequently $x \times y \times z : \overline{x+y} \times \overline{y+z} \times \overline{x+z} :: Bd \times Ce \times Af : AB \times BC \times CA :: Pe \times Pf \times Pd : AP \times BP \times CP$. *Q. E. D.*

The ſame anſwered by ΣΕΜΝΟΣ.

Let ABC be the given Triangle from whoſe Angular Points the Right Lines A*e*, B*f*, and C*d* are drawn to the oppoſite Sides; and let the Perpendiculars AO, BH, CD, *e*K, *d*I and *f*Q be alſo drawn: Then by ſimilar Triangles.

$Ce : BC :: eK : BH$
$Bd : BA :: dI : AO$
$Af : AC :: fQ : CD$
} therefore $Ce \times Bd \times Af : BC \times BA \times AC :: eK \times dI \times fQ : BH \times AO \times CD$.

In like Manner,

$Pe : PC :: eK : DC$
$Pd : PB :: dI : BH$
$Pf : PA :: fQ : AO$
} therefore $Pe \times Pd \times Pf : PC \times PB \times PA :: (eK \times dI \times fQ : DC \times BH \times AO ::) Ce \times Bd \times Af : BC \times BA \times AC$. *Q. E. D.*

Meſſ. *R*——*ſon*, *N*——*n*, and *R. R. H.* ſent Demonſtrations of this Problem.

PROBLEM XLIV. *Anſwered by* ΣΟΦΟΣ.

Let P repreſent the true Pole and F the falſe One, or that Point in the Meridian to which the Style of the Dial points, when placed in the Latitude of 54°; alſo let S repreſent the Place of the Sun: Then it is evident that the true Time will be expreſſed by the Angle SPF, and the falſe Time by the Angle SFR; conſequently the Dif-

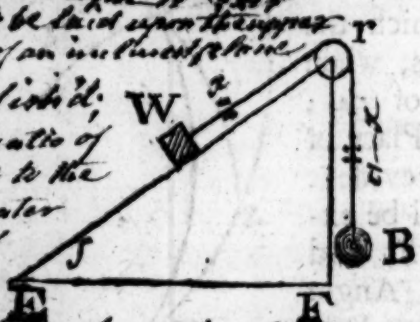
ference

ference of theſe Angles will be the Error of the Dial. Now, if *s*P be leſs than SP, the Angle *s*FR will be greater than SFR, and conſequently *s*FR—*s*PF greater than SFR—SPF: Whence, it is plain that, the leſs the Side SP, or the greater the Declination, is, the greater will be the Difference between the true and apparent Times. But the Difference between thoſe Angles is known to be the greateſt when their Fluxions are equal; and their Fluxions are known to be to each other as *Rad.* × *S.* FS : *S.* PS × *Cof.* S, univerſally. Therefore, if the Sine of PS be put $= p$, its Coſine $= d$, the Coſine of PF $= b$, that of FS $= x$, and that of S $= y$, we ſhall have $y = \frac{b - dx}{p\sqrt{1 - xx}}$, and $\sqrt{1 - x^2} = \frac{b - dx}{\sqrt{1 - xx}}$; conſequently $1 - x^2 = b - dx$, whence $x^2 - dx = 1 - b$, $x^2 - dx + \frac{dd}{4} = \frac{dd}{4} + 1 - b$, $x - \frac{d}{2} = \frac{\sqrt{dd + 4 - 4b}}{2}$, and $x = \frac{d \pm \sqrt{dd + 4 - 4b}}{2}$ $= 4007942$: Which being given, the true Time will be had $= 6$ H. 15 M. 27 S. from Noon, and conſequently the Error required $= 4$ M. 27 S.

Mr. *R*——*ſon*, alſo, anſwered this Problem.

To one end of a Cord let a Ball be ſtrung, and to the other a ſmooth ſlat weight be laid upon the ſame and ſet it, if of an inclined plane perfectly poliſhed; then if the ratio of the weight to the Ball be greater than that of Radius to the ſine of the plane's inclination, the weight will deſcend along the plane FE, and raiſe the Ball B: Now ſuppoſe the inſtant the Ball is thus made to aſcend, it is alſo put into a vibratory motion; it is required to find how many Oſcillations it will make in its whole aſcent; the inclination of the plane and length of the Cord being given.

PROBLEM XLV. *Anſwered by* ΦΙΛΟΣ.



Let Er repreſent the propoſed Plane, *W* the given Weight, *B* the Ball, and WrB the Cord by which they are connected; the Length of which let be denoted by *a*, and

and let the Sine of the Plane's Inclination, to the Radius 1, be expreſſed by s. Moreover, let the Length of a Pendulum vibrating in a Second be denoted by b, and the Diſtance deſcended freely from Reſt in the firſt Second by d, and ſuppoſe W r $= x$. Then by the Principles of Mechanics sW will be the Force by which the Weight endeavours to deſcend along the Plane; from which taking B the Force of the Ball acting in Oppoſition thereto, we have $sW - B$ for the Force by which the two Bodies are accelerated. But $W+B : sW-B :: B : \frac{sW-B \times B}{W+B}$ the Part of the Force by which B is accelerated; which is therefore to the abſolute Force acting upon B (when it deſcends freely from Reſt) as $\frac{sW-B \times B}{W+B} : B$, or as q to 1, ſuppoſing $q = \frac{sW-B}{W+B}$: Whence we have $1 : q :: d : dq$ the Diſtance moved by the Ball in the firſt Second; therefore $\sqrt{dq}$ will be the Time of aſcending through the Diſtince x, and conſequently its Fluxion $\frac{\dot{x}}{2\sqrt{dqx}}$, the Time of deſcribing $\dot{x}$.

But $\sqrt{b} : \sqrt{a-x} :: 1'' : \sqrt{\frac{a-x}{b}}$, the Time of one Vibration of the Pendulum, whoſe Length is $a-x$ (rB); and therefore $\sqrt{\frac{a-x}{b}} : 1^{vib.} :: \frac{\dot{x}}{2\sqrt{dqx}} : \frac{1}{2}\sqrt{\frac{b}{dq}} \times \frac{\dot{x}}{\sqrt{ax-xx}}$ the Vibrations per form'd in the Time of aſcending through the Diſtance $\dot{x}$; whoſe whole Fluent, when $x = a$, will

* whoſe fluent is $\frac{\sqrt{b}}{a\sqrt{dq}}$ × arch of a Circle whoſe diameter is $= a$, and verſed ſine x:

For $HC(\sqrt{ax-x^2}) : CO(\frac{a}{2}) :: Hn(\dot{x}) : hH = \frac{a\dot{x}}{2\sqrt{ax-x^2}}$ ∴ when $x = a$, the fluent becomes $\frac{\sqrt{b}}{a\sqrt{dq}}$ × the ſemiperiphery of a Circle whoſe diameter is a.

$= \frac{\sqrt{b}}{\sqrt{dq}} \times 1.5707963$ &c.

will be $\frac{p}{2}\sqrt{\frac{b}{dq}}$, p being the Length of the Arch of a Semicircle whose Radius is Unity; which Fluent (giving the Number of Vibrations required) may be expressed in a more simple Form: For, since (*by Art.* 463, *Simpson's* Fluxions) $1 : \frac{pp}{2} :: b : d$, we have $\frac{b}{d} = \frac{2}{pp}$, which Value substituted above gives $\frac{1}{\sqrt{2q}} = \sqrt{\frac{W+B}{2sW-2B}}$: From whence it appears, that the Number of Vibrations will be the same, let the Length of the Cord and the Force of Gravity be what they will.

COROLLARY.

When the Weight W descends perpendicular to the Horizon, then, s becoming $= 1$, the Number of Vibrations will be expressed by $\sqrt{\frac{W+B}{2W-2B}}$.

The same answered by the Proposer, Waltonienſis.

Let B represent the Ball, W the Weight, and s the Sine of the Plane's Inclination, Radius being Unity: Then the Force, to accelerate the Motion of the Ball along the Plane, will be to the accelerating Force of Gravity, as $\frac{sW-B}{W+B}$ to 1.

Let m denote the Space descended, in the first Second of Time, by a Body falling freely from Rest; $n\ (= \frac{sW-B}{W+B}m)$ the Space moved over by the Weight in the first Second of Time; and t the Time of the Ball's being in Motion: Now, the Space

Space moved thro' in a given Time by a Body uniformly accelerated, computing from the Beginning of the Motion, being directly as the accelerating Force, we have $m : n :: mt^2 : nt^2$, the Space moved over by the Weight in the Time t; consequently, if a be the Length of the whole Cord, $a - nt^2$ will be the Length of the vibrating Part thereof. Moreover, the Vibrations performed in the same Time, by different Pendulums, being in a reciprocal subduplicate Ratio of their Lengths, if p be the Length of a Pendulum vibrating Seconds, we shall have $\frac{1}{\overline{p|^{\frac{1}{2}}}} : \frac{1}{\overline{a - nt^2|^{\frac{1}{2}}}} :: 1_{A}^{vib.} : \frac{\overline{p|^{\frac{1}{2}}}}{\overline{a - nt^2|^{\frac{1}{2}}}}$, the Vibrations performed in a Second by a Pendulum of the Length $a - nt^2$; and $1'' : \frac{\overline{p|^{\frac{1}{2}}}}{\overline{a - nt^2|^{\frac{1}{2}}}} :: \dot{t} : \frac{\overline{p|^{\frac{1}{2}}}\,\dot{t}}{\overline{a - nt^2|^{\frac{1}{2}}}}$, the Fluxion of the Number of Oscillations made by the Ball at the End of any variable Time t; whose Fluent, when $nt^2 = a$ (*that is* when the Ball arrives at the Pulley) is $\overline{\frac{p}{n}|^{\frac{1}{2}}} \times$ Semi-circumference of a Circle whose Diameter is Unity. (1.5707963 2679)

Corollary.

The Ball, Weight, and Inclination of the Plane being given, the Number of Oscillations made by the Ball in its whole Ascent will be the same, let the Length of the Cord be what it will: And that Number will be to the Number made in the same Time,

The fluent of $\frac{p^{\frac{1}{2}}\dot{t}}{a - nt^2|^{\frac{1}{2}}} = .0174532925199432 96$

$\frac{p^{\frac{1}{2}}}{\sqrt{n}} \times$ degrees in the arc of a Circle whose radius $= 1$, and nat Sine $= \frac{nt^2|^{\frac{1}{2}}}{a}$. Which when $nt^2 = a$ that is when the Ball arrives at the Pulley becomes $\frac{p|^{\frac{1}{2}}}{n} \times 1.5707963 2679$ &c.

Time, by a Pendulum of a Length equal to that of the whole Cord, as Half the Circumference of a Circle to its Diameter.

Scholium.

If We had supposed the Cord to be drawn up by an uniform Motion, and n to be the Number of Inches *per* Second, we should have had $a - nt$ for the Length of the vibrating Part of the Cord, and $\frac{\overline{p}^{\frac{1}{2}} \dot{t}}{\overline{a - nt}^{\frac{1}{2}}}$ for the Fluxion of the Number of Vibrations: From whose Fluent, $\frac{2a^{\frac{1}{2}}p^{\frac{1}{2}}}{n}$, it appears that the Number of Oscillations made by a Ball, drawn up with any uniform Velocity, is to the Number made in the same Time by a Pendulum, equal in Length to the whole Cord, as 2 to 1.

Problem XLVI. *Answered by Mr.* R——son.

Let a be the Complement of the equal Lives, or the Number of Years betwixt the given Age and the Extremity of Old Age (which M. *D'Moïvre* places at 86); let n denote the Number of Lives and x the Number of Years required. Now, as the Probability of any one of the said equal Lives continuing to the End of x Years is expressed by $\frac{a-x}{a}$, the Probability of their All continuing in Being to the End of that Time will therefore be expressed by $\overline{\frac{a-x}{a}}^{n}$, which by the Problem is given

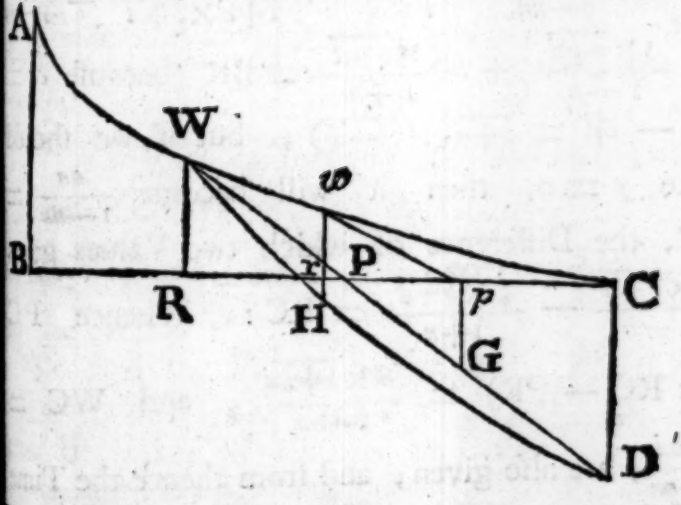
ven $= \frac{1}{2}$. Hence we have $\frac{a-x}{a} = \sqrt[n]{\frac{1}{2}}$, and

nſequently $x = a - \frac{a}{2^{\frac{1}{n}}}$.

The Propoſer ſent a curious Solution, but it me too late to be inſerted.

PROBLEM XLVII. *Anſwered by* ΦΙΛΟΞΕΝΟΣ.

It is evident that the Diſtance of the two Ships, ter the Chace begins, is not at all affected by the urrent, and that the Curve deſcribed by the Man f War on the Surface of the Water (conſidered dependently of the common Motion of the Wa- r and all Bodies in it) will be the known, or mmon *Curve of Purſuit*. Let W repreſent the



lace of the Man of War, and P that of the rivateer, at the Beginning of the Chace; then e Ordinate RW (*c*) and the Tangent WP (*b*) ing given, the Subtangent RP (*d*) will alſo be ven: And, if the Ratio of the Velocity of the an of War to that of the Privateer be as 1 to *n*, and

and the Tangent AB, which is perpendicular
the Axis BRC, be denoted by a; then (*by Art.* 4
of *Simpſon*'s Fluxions) the general Expreſſion
the Tangent will be $\frac{a^n y^{1-n} + a^{-n} y^{n+1}}{2}$, an
that of the Subtangent $\frac{a^n y^{1-n} - a^{-n} y^{n+1}}{2}$
The Sum of theſe is $a^n y^{1-n}$; which, when
$(=WR) = c$, becomes $a^n c^{1-n} = b+d$; whenc
$a^n = \overline{b+d} \times c^{n-1}$, from which the Value of a
given.

Moreover the general Value of any Abſciſſa (B
being $\frac{an}{1-nn} - \frac{\frac{1}{2}a^n y^{1-n}}{1-n} + \frac{\frac{1}{2}a^{-n} y^{1+n}}{1+n}$, by ſub
ſtituting therein for a^n, and taking $y=c$, it wi
become $\frac{an}{1-nn} - \frac{\frac{1}{2}\times\overline{b+d}}{1-n} + \frac{\frac{1}{2}ec}{1+n\times\overline{b+d}} = \frac{an}{1-n}$
$- \frac{\frac{1}{2}\times\overline{b+d}}{1-n} + \frac{\frac{1}{2}\times\overline{b-d}}{1+n} = BR$ (becauſe $ct =$
$bb - dd = \overline{b+d} \times \overline{b-d}$); but if we ſhoul
take $y = o$, then it will become $\frac{an}{1-nn} =$
BC, the Difference of which two Values give
$\frac{\frac{1}{2}\times\overline{b+d}}{1-n} - \frac{\frac{1}{2}\times\overline{b-d}}{1+n} = RC$: Whence PC
$(= RC - PR) = \frac{n\times\overline{b+nd}}{1-nn}$, and WC =
$\frac{b+nd}{1-nn}$, are alſo given; and from thence the Time
required = 6H. 41M. 31S.

But to conſtruct the Curve which the Man of
War deſcribes in abſolute Space, including the
Motion of the Current, having determined the
Line PC and the cotemporary Poſitions w, p, as
above, let pG be parallel to wr, meeting WP
produced

produced in G, and in *wr* produced take WH = *r*G; then it is evident that the Curve WHD paſſing thro' all the Points H, thus determined, will be That required.

The Propoſer, alſo, anſwered this Problem; and Mr. *R——ſon* anſwered it upon the ſame Principles as the above.

PROBLEM XLVIII. *Anſwered by R. R. H.*

Let the Sine and Coſine of the Arch *z*, be denoted by *y* and *u*, reſpectively; then by the Nature of the Circle $ry = u\dot{z}$, and $y\dot{z} = -r\dot{u}$: Now, by the Property of the Curve, $\frac{z^n}{r^{n-1}} \times -\dot{u}$ will be the Fluxion of the Area; whoſe Fluent, by aſſuming $z^n u + s$ for the Fluent of $z^n\dot{u}$, $s = t - rnz^{n-1}y$, $t = n.\overline{n-1}.r^2z^{n-2}u + v$, &c. will appear to be $= -r^{1-n}z^n u + nyr^{2-n}z^{n-1} + n.\overline{n-1}.ur^{3-n}z^{n-2} - n.\overline{n-1}.\overline{n-2}.yr^{4-n}z^{n-3}$, &c. the Area required.

The ſame anſwered by Mr. R——ſon.

Let *y* be the Sine and *x* the Coſine of the Arc *z*, to the Radius *r*; then $\frac{-z^n\dot{x}}{r^{n-1}}$ will be the Fluxion of the Area.

To find the Fluent hereof let it be aſſumed equal to $Az^n + Bz^{n-1} + Cz^{n-2} + Dz^{n-3}$ &c. and then by taking the Fluxion we have

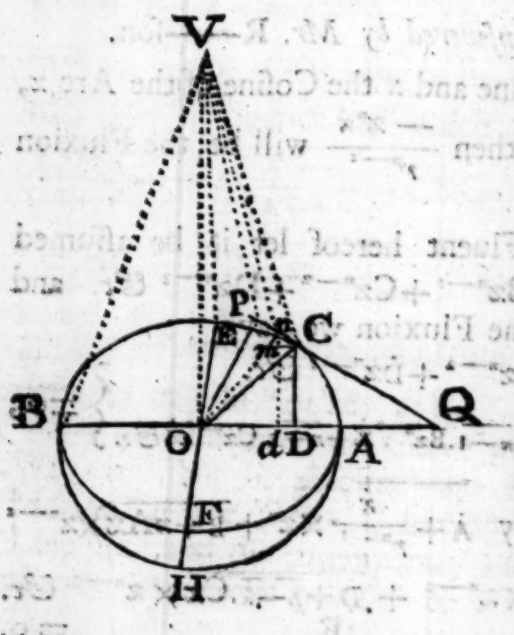
$$\left.\begin{array}{l} \dot{A}z^n + \dot{B}z^{n-1} + \dot{C}z^{n-2} + \dot{D}z^{n-3} \text{ \&c.} \\ + \frac{z^n\dot{x}}{r^{n-1}} + nAz^{n-1}\dot{z} + \overline{n-1}.Bz^{n-2}\dot{z} + \overline{n-2}.Cz^{n-3}\dot{z} \text{ \&c.} \end{array}\right\} = 0;$$

and conſequently $\dot{A} + \frac{\dot{x}}{r^{n-1}} \times z^n + \overline{B + nA\dot{z}} \times z^{n-1} + \overline{C + n-1.B\dot{z}} \times z^{n-2} + \overline{D + n-2.C\dot{z}} \times z^{n-3}$ &c.

$= 0.$

$= o$. Whence, by making $\dot{A} + \frac{\dot{x}}{y^{n-1}} = o$, $\dot{B} + nA\dot{z}$ $= o$, $\dot{C} + \overline{n-1}.B\dot{z} = o$, &c. we get $\dot{A} = -\frac{\dot{x}}{y^{n-1}}$, $\dot{B} = \frac{nx\dot{z}}{y^{n-1}} = \frac{n\dot{y}}{y^{n-2}}$ (becauſe $\dot{z} = -\frac{r\dot{y}}{x}$) $B = \frac{ny}{y^{n-1}}$, $\dot{C} = -\overline{n-1} \times B\dot{z}$, $C = \frac{n.\overline{n-1}.x}{y^{n-3}}$, $\dot{D} = -\overline{n-2}.C\dot{z}$, $D = -\frac{n.n-1.n-2.y}{y^{n-4}}$, &c. Therefore the Area ſought is truly defined by $-\frac{xz^n}{y^{n-1}} + \frac{nyz^{n-1}}{y^{n-2}} + \frac{n.n-1.xz^{n-2}}{y^{n-3}} - \frac{n.n-1.n-2.yz^{n-3}}{y^{n-4}}$, &c. Where the Law of Continuation is manifeſt, and where the Series will always terminate when n is a whole poſitive Number.

PROBLEM XLIX. *Anſwered by the Propoſer,* ΣΕΜΝΟΣ.



Let V be the Vertex of the Cone, and AEBF the Baſe thereof, to which let PCQ be a Tangent at any Point C: Moreover let CD be an Ordinate to the Tranſverſe Axis AB, and ſuppoſing OP perpendicular to QCP, and *dn* as near as poſſible to DC, let OC,

On, VC, Vn and VP be drawn: Putting the Semi-tranſverſe Axis AC $= a$, the Semiconjugate OE $= b$, the Altitude of the Cone OV $= c$, AD $= x$, DC $= y$, $Cz = z$, $Dd = \dot{x}$, &c.

Then, by the Property of the Curve OQ will $z = \frac{aa}{x}$; and by ſimilar Triangles Cn ($\dot{z}$) : mn ($-\dot{y}$) :: OQ $\left(\frac{aa}{x}\right)$: OP $= -\frac{aa\dot{y}}{xz}$, whence PV $(= \sqrt{OP^2 + OV^2}) = \sqrt{\frac{a^4\dot{y}^2}{x^2z^2} + c^2}$, and conſequently the Fluxion CVn of the Surface $= (\frac{1}{2}Cn \times VP =) \frac{\sqrt{a^4\dot{y}^2 + c^2x^2\dot{z}^2}}{2x} = \frac{\sqrt{a^4 + c^2x^2 \times \dot{y}^2 + c^2x^2\dot{x}^2}}{2x}$, becauſe $\dot{z}^2 = \dot{x}^2 + \dot{y}^2$. Now, by the Property of the Curve, $y = \frac{b}{a}\sqrt{a^2 - x^2}$, and ſo $\dot{y} = \frac{-bx\dot{x}}{a\sqrt{a^2 - x^2}}$, which being ſubſtituted above, the Fluxion of the Surface becomes $\frac{\dot{x}}{2\sqrt{a^2 - x^2}} \times \sqrt{a^2 \times b^2 + c^2 - 1 - \frac{bb}{aa} \times c^2x^2}$. Put $\sqrt{bb + cc} (= VE) = g$, and $1 - \frac{bb}{aa} \times \frac{cc}{gg} = d$, then it will become $\frac{g\dot{x}}{2\sqrt{aa - xx}} \times \overline{a^2 - dx^2}^{\frac{1}{2}}$, whoſe Fluent (*by Art.* 434 of *Simpſon*'s Fluxions) when x is $= a$, will be $\frac{pag}{4}$ × into $1 - \frac{d}{2.2} - \frac{3.dd}{2.2.4.4} - \frac{3.3.5.ddd}{2.2.4.4.6.6} - \frac{3.3.5.5.7.dddd}{2.2.4.4.6.6.8.8.}$, &c. the Quadruple of which is the whole Surface required.

COROLLARY 1.

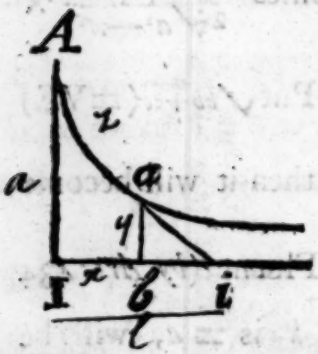
If $c=0$, then $d=0$, $g=b$, and the Surface itſ
(which in this Caſe is the Elliptical Baſe) will b
$=pab$.

COROLLARY 2.

If AHB be another Semi-Ellipſis on the ſam
Tranſverſe Axis AB, whoſe Focal Diſtance is t
That of the given Ellipſis, as the perpendicula
Height of the Cone to the Length of the Lin
(VE), drawn from the Vertex to the Extremity o
the Conjugate Diameter; then it is evident, fro
the Place above quoted, that the required Surfac
of the Cone will be equal to the Rectangle unde
the ſaid Line VE and the Semi-Elliptical Arc
AHB.

Mr. *R——ſon* ſent a Solution to this Problem
in the ſame Manner as above.

PROBLEM L. *Anſwered by the Propoſe*
Waltonienſis.



Let A and I repreſent th
Places of the Pyrate an
Indiaman at the Beginning o
the Chace, and *a* and *i* tw
other cotemporary Poſition
of them, whilſt the Forme
is purſuing the Latter.

Then, calling I*b*, x; *a*
paralel to AI, y; the Curv
A*a*, z; I*i*, l; and AI, a; it is $y : a :: -\dot{y}$
$\frac{-a\dot{y}}{y} = \dot{z}$. Therefore Hyp. Log. of $\frac{aa}{yy} = z$
and, $\sqrt{aa-yy}$ being $= bi$, $y : \sqrt{aa-yy} :: -\dot{y} :$
$\sqrt{aa-yy} \times \frac{\dot{y}}{y} = \dot{x}$.

Now

Now $\frac{-yy\dot{y}}{\sqrt{aa-yy}}$, the Fluxion of $\sqrt{aa-yy}$, added to $-\sqrt{aa-yy} \times \frac{\dot{y}}{y}$, the Fluxion of x, gives $\frac{-aay^{-1}\dot{y}}{\sqrt{aa-yy}} = i$: Whence $a \times$ Hyp. Log. $\frac{y}{a-\sqrt{aa-yy}} = l$; and, n being the Number whoſe Hyperb. Log. is l, y is found $= \frac{2an^{\frac{1}{a}}}{n^{\frac{2}{a}}+1}$; conſequently $a \times$ Hyp. Log. $\frac{n^{\frac{2}{a}}+1}{2n^{\frac{1}{a}}}$ is the Diſtance the Pyrate ſailed during the Chace.

The ſame anſwered by Mr. Leigh.

Let AI $= ai$ be denoted by a, and let the Diſtance run by the Merchantman, in the Time of the Purſuit, be denoted by b; then if bi be expreſſed by y, ab will be expreſſed by $\sqrt{aa-yy}$. Therefore $\sqrt{aa-yy} : a :: \frac{yy\dot{y}}{\sqrt{aa-yy}} : \frac{ayy\dot{y}}{aa-yy}$ the Fluxion of the Curve; whence the Fluxion of Ib will be had $= \frac{yy\dot{y}}{aa-yy}$, and conſequently the Fluxion of $Ii = \frac{aa\dot{y}}{aa-yy} = \frac{1}{2}a \times \frac{2a\dot{y}}{aa-yy}$; whoſe Fluent will be repreſented by $\frac{1}{2}a$ into the Hyp. Log. $\frac{a+y}{a-y} = b$: Whence y will be found $= 1.999$, and from thence the Length of the Curve, or $\frac{1}{2}a$ into the Hyp. Log. $\frac{aa}{aa-yy}$ will be had $= 8.61379$ the

the Diſtance run by the Pyrate in the Time of the Purſuit.

Meſſ. *R*——*ſon*, *N*——*n*, and *R. R. H.* ſent Solutions to this Problem.

PROBLEM LI. *Anſwered by Mr.* R——ſon.

The given Expreſſion $\frac{a^6\dot{x}}{a^6+a^4x^2+a^2x^4+x^6}$ is $=$ $\frac{\frac{1}{2}a^2\dot{x}}{a^2+x^2}+\frac{\frac{1}{2}a^2\dot{x}\times\overline{aa-xx}}{a^4+x^4}$, and a^4+x^4 is $=$ $\overline{aa+ax\sqrt{2}+xx}\times\overline{aa-ax\sqrt{2}+xx}$. Put $a\sqrt{2}=c$, and aſſume $\frac{A+Bx}{aa+cx+xx}+\frac{C+Dx}{aa-cx+xx}=\frac{aa-xx}{a^4+x^4}$, then

$$\left.\begin{array}{rrrr} Aaa & - Acx & + Axx & \\ & + Baax & - Bcxx & + Bx^3 \\ Caa & + Ccx & + Cxx & \\ & + Daax & + Dcxx & + Dx^3 \\ -aa & & + xx & \end{array}\right\} = 0:$$

Whence, by comparing the homologous Terms, we have $A=\frac{1}{2}$, $B=\frac{1}{c}$, $C=\frac{1}{2}$, and $D=-\frac{1}{c}$; and therefore $\frac{\frac{1}{2}+\frac{x}{c}}{aa+cx+xx}+\frac{\frac{1}{2}+\frac{x}{c}}{aa-cx+xx}=\frac{aa-xx}{a^4+x^4}$, and $\frac{\frac{1}{2}aa\dot{x}\times\overline{aa-xx}}{a^4+x^4}=\frac{aa}{4c}$ into $\frac{c\dot{x}+2x\dot{x}}{aa+cx+xx}+\frac{c\dot{x}-2x\dot{x}}{aa-cx+xx}$: Whoſe Fluent, it is evident, will be $\frac{aa}{4c}$ into the Hyp. Log. $aa+cx+xx$ — Hyp. Log. $aa-cx+xx=\frac{a}{4\sqrt{2}}$ into the Hyp. Log. $\frac{aa+ax\sqrt{2}+xx}{aa-ax\sqrt{2}+xx}$; to which if the Fluent of the

firſt

rm, or Half the Arch of the Circle whoſe dius is *a* and Tangent *x*, be added, you will ve the whole Fluent ſought.

Meſſ. *R. R. H.* and *N*——*n* pointed out Methods anſwer this Problem.

PROBLEM LII. *Anſwered by the Propoſer.*

Let 84¼ Minutes, the Time of Revolution at e Earth's Surface, be denoted by *q*; 180 Minutes, e given Time, by *s*; and let *t* be the Tangent an Arch *A* whoſe Radius is 1: Then, by Art. 99 of *Simpſon*'s Fluxions, we ſhall have $\overline{1+tt}^{\frac{1}{2}} \times \frac{+tt \times A - 3t}{2t^3} = \frac{qq}{3ss}$; whence *t* will be had =.879, $B\ (=t^2) = .7726$, and the Time ſought $\overline{1+B}^{\frac{1}{2}} \times s) = 198$ Minutes. Therefore, if 000 Miles, the Diameter of the given Sphere, be enoted by *a*, and $\sqrt{1+B}\ (=1.331)$ by *n*, alſo he Polar Axis by *x*, we ſhall have *nx* for the quatoreal Diameter, and $n^2x^3 = a^3$; whence $x = \frac{a}{n^{\frac{2}{3}}}$ will be given = 6610, and the Equatoreal Diameter = 8798.

A COL-

A COLLECTION *of* PROBLEMS *to be answered in the next* NUMBER.

PROBLEM LIII. *By* Lupus.

THERE are two Numbers, ſuch that the Leſſer is to the Greater, as 1 to 8; and their Sum is to their Product, likewiſe, as 1 to 8. What are the Numbers?

PROBLEM LIV. *By* ΣΟΦΟΣ.

Out of a Baſket of Apples, A took 5; B, 7; and C, 10; then A took Half what were left, and the Reſt were divided between B and C; ſo that, at laſt, They all Three had an equal Share. How many Apples had Each?

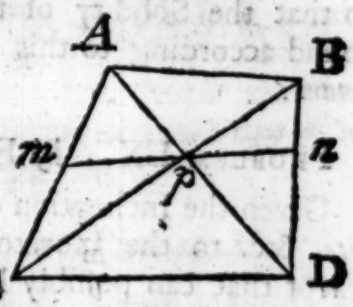
PROBLEM LV. *By* Proteus.

A Gentleman employ'd a certain Number of Workmen in making a Fiſh-pond: When they had been at Work 24 Days, and had half finiſhed, He ſet on 16 Men more; by which Means the remaining Half of the Work was compleated in 16 Days: The Queſtion is, to find the Number of Men employ'd at firſt, and the whole Expence at 1 *s.* 6 *d.* a Day each Man.

PROBLEM

Problem LVI. *By* Waltonienſis.

In the Trapezium ABCD, if any Right-Line *mn* be drawn through *p*, the Point where the Diagonals interſect each other; A*m*×B*n* will be to C*m*×D*n*, as A*p*×B*p* to C*p*×D*p*: Quere the Demonſtration?



Problem LVII. *By* ΣΕΜΝΟΣ.

The Baſe and Perpendicular of a plane Triangle being given, to determine the Sides; ſo that the Ratio of the Greater to the Leſſer ſhall be a *Maximum*.

Problem LVIII. *By Mr.* William Toft.

A Line being drawn from any Point in the Circumference of a given Circle, making a given Angle with the Diameter drawn from that Point; 'tis propoſed to find a Point in the Periphery of the Circle, ſo that a Line being drawn from it to the firſt Point and Another, in a given Ratio thereto, terminating in the firſt Line, the Part of the ſaid firſt Line, intercepted by theſe two Lines, ſhall be a *Maximum*.

Problem LIX. *By the ſame.*

In the practical Menſuration of a Conical Piece of Timber, it is cuſtomary to girt it in the Middle, and take one Fourth of the Girt as the Side of a Square; and the Area of this Square, multiplied by the Length of the Timber, is taken for the ſolid Content. Now, if there be ſuppoſed a

Cone of a given Baſe and Height, 'tis required t find what Part muſt be cut off next the Verte ſo that the Solidity of the remaining Fruſtrum found according to this Method, may be a *Max imum?*

PROBLEM LX. *By* Een Bouw-man. * * *

Given the Inclination of the Plane of a *Theodo lite*, &c. to the Horizon; required the greate Error that can poſſibly happen in taking the Va lue of an Angle?

PROBLEM LXI. *By* Curioſus.

To determine the Poſition of the Moon's Node ſo that, when her Declination is the greateſt poſſi ble, her Right Aſcenſion may alſo be a *Maximum.*

PROBLEM LXII. *By the ſame.*

The Declination of a Wall, in the Lat. of 51 32′ N. being 24 Degrees Weſterly; 'tis require to find at what Hour of the Day, on *Jan.* 10 N. S. its Shadow will be the ſhorteſt poſſible?

PROBLEM LXIII. *By* Honeſtus.

To find an Equation for the Relation of x an y, ſuppoſing $\frac{2\dot{x}}{x}+\frac{3\dot{y}}{y}=\frac{x^{1}\dot{y}}{ay^{3}}$, and that when $x=a$ y is alſo $=a$.

PROBLEM LXIV. *By* ΦΙΛΟΣ.

Suppoſing there to be three Veſſels, full of Li quor, under the different Forms of a Cylinder, a Hemiſphere, and a Cone, whereof the Baſes, an Altitudes, are all equal; 'tis required to find th Ratio of the Times in which They will be evacuate at equal Holes in their Baſes?

PROBLE

PROBLEM LXV. *By Mr.* W. Toft.

To find the hotteſt Time of the Day in any given Latitude; ſuppoſing the Heat to be conjunctly as any given Power of the Sine of the Sun's Altitude, and any given Power of the Time of his Continuance above the Horizon.

PROBLEM LXVI. *By* Waltonienſis.

Suppoſing 3 Balls A, B, C, whoſe Weights are 1, 2, and 3 Pounds reſpectively, to be connected together by a ſtraight inflexible Rod of a given Length; A being faſtned at one End, B in the Middle, and C at the other End; and ſuppoſing the Ball A to be ſtruck by a fourth Ball D, of 4 Pounds Weight, moving on an Horizontal-plane (on which A, B and C are to be ſuppoſed to reſt) with a Celerity *c*, in a Direction at Right Angles to the Rod: It is required to determine the Motion of each of the Balls after the Impulſe, conſidering them all as perfectly hard.

PROBLEM LXVII. *By* ΦΙΛΟΞΕΝΟΣ.

To determine the Momentum of Rotation of an oblate Spheroid, revolving about an Equatoreal Diameter at reſt.

Note, This Problem is of Uſe in finding the Preceſſion of the Equinox.

MEMORIAL VERSES

Adapted to the Gregorian Account, *or* New Style.

Thoſe mark'd with an Aſteriſm by *J. Canton*, M. A. and F. R. S. to whom we are obliged for this Paper.

* *To know if it be* LEAP YEAR.

LEAP YEAR is given, when four will divide
The Cent'ries compleat, or odd Years beſide.

Example for 1752. 4)52(0, Leap Year.
13

Example for 1800. 4)18(2, not Leap Year.
4

* *To find the* DOMINICAL LETTER.

DIVIDE the Cent'ries by 4; and twice what does remain
Take from 6; and then add to the Number you gain
The odd Years and their 4th; which dividing by 7,
What is left take from 7, and the Letter is given.

Example for 1752. 4)17(1
4
2
6
4
52
13
7)69(9
1=A.

By

By the Dominical Letter, *to find on what Day of the Week any Day of the Month will fall throughout the Year.*

At Dover Dwells GEORGE BROWN, Eſquire,
Good CHRISTOPHER FINCH, And DAVID FRIER.

Example for May 9, 1752. A being the Dominical Letter.
1 May = B = Monday.
7
8 = Monday.
1
9 = Tueſday.

To find the Golden Number, Cycle of the Sun, *and* Roman Indiction.

WHEN 1, 9, 3, to the Year have added been,
Divide by Nineteen, Twenty-eight, Fifteen.

Exaples for 1752.

1752	1752	1752
1	9	3
19)1753(92	28)1761(62	15)1755(116
43	81	25
5=G.N°.	25=Cy.S.	105
		15=R.Ind.

* *A general Rule for the* EPACT.

LET the Cent'ries by 4 be divided; and then
What remains multiply'd by the Number 17;
Forty three times the Quotient, and 86 more
Add to that; and dividing by 5 and a Score;
From 11 times the Prime, ſubtract the laſt Quote,
And rejecting the Thirties, gives th' Epact you ſought.

Example

Example for 1752.

```
4) 17 ( 1        G, N°. = 5
   — 17                11
     4                 —
    43                 55
   ———                 11
   172                 —
   86                30) 44 (1
   17                    14 = Epact.
   ——
  25) 275 (11
      25
```

* *To find the* EPACT *till the Year* 1900.

THE Prime wanting one, multiply'd by 11,
And the Thirties rejected, the Epact is given.

Example. G. N°. = 5

```
                 1
                 —
                 4
                11
                —
           30) 44 (1
               14 = Epact.
```

* *To find Easter Limit, or the Day of the Paschal Full Moon from* March *the 1st, inclusive.*

ADD six to the Epact, reject 3 times 10,
What's left take from 50, the Limit you gain:
Which if 50, one less you must make it, and even
When 49 too, if Prime's more than 11.

Example. Epact = 14

```
          6
          —
         20
         50
         —
         30 = Limit.
```

To

To find EASTER-DAY.

FROM Limit take the Letter, adding four,
From the next Sevens what's left ~~ſtill~~ take away,
What ſtill remains, with Limit found before,
Gives Eaſter-Sunday from St. David's Day.

Example. Limit = 30 A = 1
5 4
25 5
28 = next Sevens.
3
30 = Limit.
33
31 = March.
April 2 = Eaſter-Day.

To find the Age, or Change of the Moon.

JANUS 0, 2, 1, 2, 3, 4, 5, 6,
8, 8, 10, 10, theſe to the Epact fix,
The Sum, bate 30, to the Month's Day add,
Or take from 30, Age, or Change, is had.

Example. March 10, 1752. Ep. = 14
1 = N° of the Month.
15
10 = Day of the Month.
25 Days = Moon's Age.

30
15
15 March = Change.

* To

* *To find the Time of the Moon's coming to the South, and of High Water at* London-Bridge.

FOUR times the Moon's Age, if by 5 you divide,
Gives the Hour of her Southing: Add 2 for the Tide.

Example. Moon's Age = 9 Days.

4

5)36(7 H.

1

12 M. = ⅕ H.

7 H. 12 M. p. m. = Southing.

2

9 12 = High-Water.

The Problem, by *R. R. H.* we would have inſerted, but it does not ſeem to have been ſufficiently conſidered; ſince it is certain that the Moment the Plane is put in Motion the Ball will leave the Plane; as the Centrifugal Force is then greater than that Part of the Force of Gravity, by which it is urged down the Plane.

We are obliged to let the Remainder of *Honeſtus*'s Letter lye, for want of Room, till another Opportunity.

MATHEMATICAL EXERCISES:

CONTAINING,

The Principles of Dialling.

I. Anſwers to the Problems propoſed in the fourth Number.

III. Some new Forms of Fluents.

IV. A Collection of new Problems.

V. Remarks on the *Palladium*, &c.

By JOHN TURNER.

N°. V.

LONDON:

Printed for JAMES MORGAN, at the *Three Cranes* in *Thames-ſtreet*, 1752, where the preceding Numbers may be had.

[Price ONE SHILLING.]

ERRATA.

PAGE 137, Line 4, for *ſtill* read *laſt*
Page 165, Line 15, dele the *Apoſtraphe* in *Author's*.

ADVERTISEMENT.

ALL Perſons inclining to encourage this Work, are deſired to ſend their Performances (whether new Problems, Paradoxes, Solutions, &c.) Poſt paid, to be left with Mr. JAMES MORGAN, at the *Three Cranes* in *Thames-Street*, before the 25th of *April* 1753.

ANSWERS *to the* PROBLEMS *propoſed in the* FOURTH NUMBER.

PROBLEM LIII. *Anſwered by Mr.* J. Noorthouck.

LET y be the leſſer Number, then as $1 : 8 :: y : 8y$ the greater Number; and as $1 : 8 :: 9y$ (the Sum of the two Numbers) $: 8y^2$ (their Product) : Whence $8y^2 = 72y$, and $y = 9$.

The ſame anſwered by Mr. Widd.

Suppoſe x to denote the leſſer Number, then $8x$ will be the Greater, $9x$ their Sum, and $8x^2$ their Product; whence, by the Queſtion, $72x = 8x^2$, and $x = 9$ the leſſer Number ſought; conſequently $8x = 72$ the greater Number.

This Problem was, alſo, anſwered by Meſſ. *Trott, George, Brownbridge, Marſhall,* and *E. B—M.*

PROBLEM LIV. *Anſwered by Mr.* Phil. George, *of the* Iſle of Wight.

Let x repreſent the Number of Apples in the Baſket, then $x - 5 - 7 - 10$ will be the Quantity left after their firſt Draught; but as each had an equal Share at laſt, and as $\frac{x-5-7-10}{2} + 5$ was A's Share at that Time, we have $\frac{x+5-7-10}{2} = \frac{x}{3}$; whence $x = 36$, and each Man's Share $= 12$.

The ſame anſwered by Mr. E. B—M.

Let x be the Number of Apples each Perſon had, a, b and c what each took out of the Baſket, $n=3$ and $m=2$, then by the Problem $a+\frac{nx-a-b-c}{m}=x$; which Equation, reduced, gives $x=\frac{a+b+c-am}{n-m}=12$.

Meſſ. *Noorthouck*, *Widd*, *Trott*, *Moſs*, *Brownbridge* and *Marſhall* ſent Solutions to this Problem.

PROBLEM LV. *Anſwered by Mr.* Widd.

Suppoſe the Number of Men employed at firſt to be denoted by x, then by the Rule of Three Inverſe $x:24::x+16:16$; whence $24x=16x+256$, and $x=32$; conſequently the whole Expence is 115 *l.* 4 *s.*

The ſame anſwered by Mr. Noorthouck.

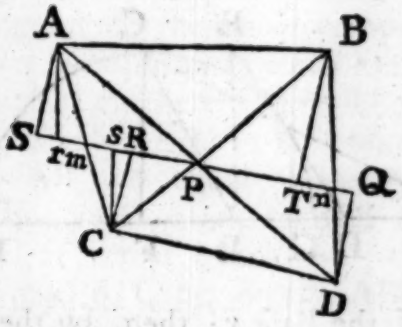
Let x be the Number of Men firſt ſet to Work, and $16+x$ the full Complement ſet to Work when Half was done; then, it being evident that one Half of the Work required as much Labour as the other, if each Number of Men be multiplied by the Days they reſpectively worked, we ſhall have $24x=256+16x$, and $x=32$.

This Problem was, alſo, anſwered by Meſſ. *E. B——M*, *Trott*, *George*, *Brownbridge*, and *Marſhall*.

PROBLEM

PROBLEM LVI. *Anſwered by Mr.* Ghis.

Upon *m n* let fall the Perpendiculars C R, AS, BT, and DQ; then, by ſimilar Triangles



$\left.\begin{matrix} Cm : Am :: CR : AS \\ Dn : Bn :: DQ : BT \end{matrix}\right\}$ and $\left.\begin{matrix} Cp : Bp :: CR : BT \\ Dp : Ap :: DQ : AS \end{matrix}\right\}$

Therefore by Compoſition and Equality $Cm \times Dn : Am \times Bn$ (:: $CR \times DQ : BT \times AS$) :: $Cp \times Dp : Bp \times Ap$. *Q. E. D.*

N. B. We forgot to mention in our laſt, that Mr. *Ghis* conſtructed the 41ſt Problem.

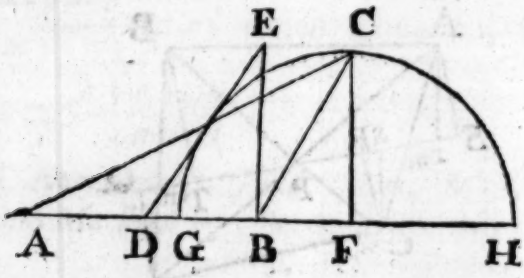
The ſame anſwered by Lupus.

Draw A *r* and C *s* parallel to BD: Then, ſince the Triangles A *rm*, A *rp*, and B *np* are ſimilar to the Triangles C *sm*, D *np*, and C *sp* reſpectively, we have $\left.\begin{matrix} Am : Ar :: Cm : Cs \\ Bn : Bp :: Cs : Cp \\ Ar : Ap :: Dn : Dp \end{matrix}\right\}$: Whence, by Compoſition and Alternation, $Am \times Bn : Cm \times Dn :: Ap \times Bp : Cp \times Dp$. *Q. E. D.*

Waltonienſis and Mr. *R——ſon*, alſo, demonſtrated this Problem.

PROBLEM LVII. *Anſwered by Mr.* Walter Trott.

For the Perpendicular put p, the Baſe b, and the Diſtance of the Perpendicular from the fartheſt



Extremity of the Baſe x; then, by the Problem, $\frac{x^2+p^2}{b^2-2bx+x^2+p^2} = \frac{AC^2}{BC^2}$ muſt be a Maximum; which put into Fluxions and reduced gives $x^2-bx = p^2$: Whence $x = \frac{b+\sqrt{b^2+4p^2}}{2}$; conſequently the Angle oppoſite to the greateſt Side is obtuſe.

Meſſ. *Widd*, *R——ſon*, *George*, *Moſs*, and *Harpſwellienſis* anſwered this Problem much after the ſame Manner.

The ſame anſwered by ΣΕΜΝΟΣ.

Let ABC be the Triangle required, and let HCG be a Circle paſſing thro' C and dividing AB in G, ſo that AG : GB : : AC : CB; then, F being the Center thereof, it will be AG : GB : : AC : CB : : AF : GF (FC) : : FC : FB. but the Ratio of AC to CB is the greateſt when That of AF to FC is the greateſt, that is when $\frac{FC}{AF}$ is a Minimum, or when FC is equal to the given Perpendicular; therefore we have AG

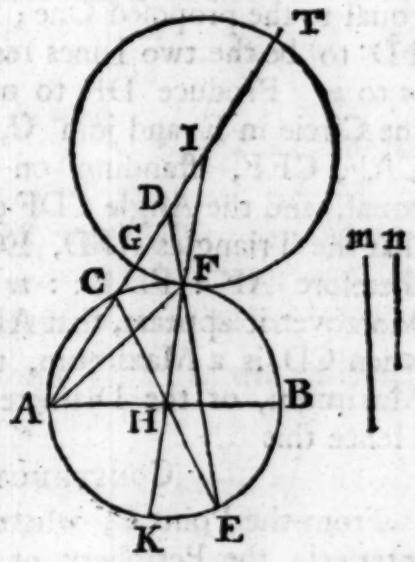
AG : GB : : AC : CB : : AF : FC : : FC : : FB, and $AF \times BF = FC^2$ = the Square of the given Perpendicular: Whence the Problem is reduced to This (being the ſame as That in pag. 94, *Simpſon*'s *Exerciſes*)

To a given Line, to add another Line, ſo that the Rectangle under the whole, compounded, Line, and the Part added, may be equal to a given Square. The Conſtruction whereof is as follows.

Having biſected AB, in D, make BE perpendicular to AB and equal to the given Perpendicular; alſo join D, E and take DF equal thereto; then if FC be drawn equal and parrallel to BE and A, C and B, C be joined, ABC will be the Triangle required.

Mr. *Moſs* ſent a Geometrical Conſtruction to this Problem.

PROBLEM LVIII. *Anſwered by Mr.* R——ſon.



Let ACBE be the given Circle whoſe Radius is r, and AT the Line given in Poſition interſecting the ſame in C; ſuppoſe AC $= g$, and let m and n expreſs the given Ratio: Then if ſuch a Diſtance AD be taken on AT that, being divided in G in the Ratio of m to n and a Circle deſcribed according to Theor. 15, Book 4, *Simpſon*'s *Geometry*, it may touch the given Circle ACBE,

it

it will, it is manifeſt, be the Maximum required. Suppoſe therefore the ſaid Circle to be deſcribed and thro' the Centers I, H draw the Right-Line IHK to meet the Periphery of the given Circle in K, then it is manifeſt the ſaid Line will paſs thro' F the Point of Contact, and IA × IC will be = IK × IF, that is, ſuppoſing AD = x, $\frac{m^2x}{m^2-n^2} \times \sqrt{\frac{m^2x}{m^2-n^2}} - g = \frac{mnx}{m^2-n^2} + 2r \times \sqrt{\frac{mnx}{m^2-n^2}}$; whence $x = \frac{2nr}{m} + g$.

Mr. *Widd* ſolved this Problem from the Principles of Fluxions.

The ſame anſwered by J. T——r.

Let ACBE be the given Circle and ACD a Line making an Angle, with the Diameter AB, equal to the propoſed One; alſo ſuppoſe AF and FD to be the two Lines required in the Ratio of m to n. Produce DF to meet the Periphery of the Circle in E and join C, E; ſince the Angles CAF, CEF, ſtanding on the ſame Arch, are equal, and the Angle CDF common, it is evident that the Triangles AFD, ECD are ſimilar, and therefore AF : FD (: : m : n) : : EC : CD. Moreover it appears, that AD will be a Maximum when CD is a Maximum, that is, when CE is a Maximum, or the Diameter of the given Circle. Hence this

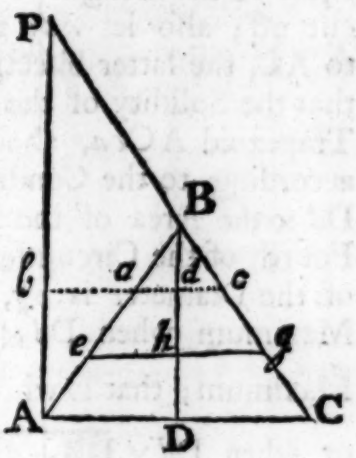
CONSTRUCTION.

From the Point C, where the propoſed Line interſects the Periphery of the Circle, draw the Diameter CE and take CE : CD : : m : n; then join D, E and from the Point F, where it in-

ſects

terſects the Circumference, draw AF, and the Thing is done.

PROBLEM LIX. *Anſwered by Mr.* Moſs.



Let ABC be a Triangular Section of the Cone thro' the Center of its Baſe, and BD the Altitude thereof, parallel to which draw AP meeting CB produced in P: Alſo let *a*B*c* be the Section of the Part to be cut off; then $bc\left(=\frac{AC+ac}{2}\right)$ will be = Half the Sum of the Diameters AC and *ac*; and if AP = 2BD be put = *a*, AC = *b*, and *bc* = *x*, we ſhall have, by ſimilar Triangles, $b : a :: x : \frac{ax}{b} = Pb$; whence $Ab = a - \frac{ax}{b}$, and $\frac{ba-ax}{b} \times px^2$ (*p* being =.7854) the Content of the Fruſtum, according to the Method propoſed, which is to be a Maximum; therefore $bax^2 - ax^3$ a Maximum and its Fluxion $2bax\dot{x} - 3ax^2\dot{x}$ or $2b - 3x = 0$, whence $x = \frac{2b}{3}$, and conſequently $Ab = \frac{1}{3}$ of the Altitude of the Cone.

This Problem was, alſo, anſwered by Meſſ. *Widd, E. B—M, Trott, R——ſon, George,* and *Brownbridge.*

The

The ſame anſwered by Proteus.

Let ABC be a Triangular Section of the Cone through the Center of its Baſe, and let B*d* be equal to the Length of the Axis of the Part to be cut off; alſo let *adc* and *ebg* be drawn parallel to AC, the latter biſecting D*d* in *b*. It is evident that the Solidity of the Fruſtum deſcribed by the Trapezoid A*Cca*, about D*d* as an Axis, will be, according to the Conditions of the Problem, as D*d* × the Area of the Square, whoſe Side is one Fourth of the Circumference of the Circle whereof the Diameter is *eg*, and therefore muſt be a Maximum when D*d* × Area of that Circle is a Maximum; that is when $Dd \times \overline{be}^2$ is a Maximum, or when $Dd \times \overline{DB+dB}^2$ is a Maximum.

But it may be eaſily demonſtrated that $Dd \times \overline{DB+dB}^2$ will be a Maximum when B*d* is = $\frac{1}{3}$ DB. Whence it follows that the Solidity of the Fruſtum will be the greateſt when the Altitude thereof is equal to two Thirds of the Altitude of the whole Cone.

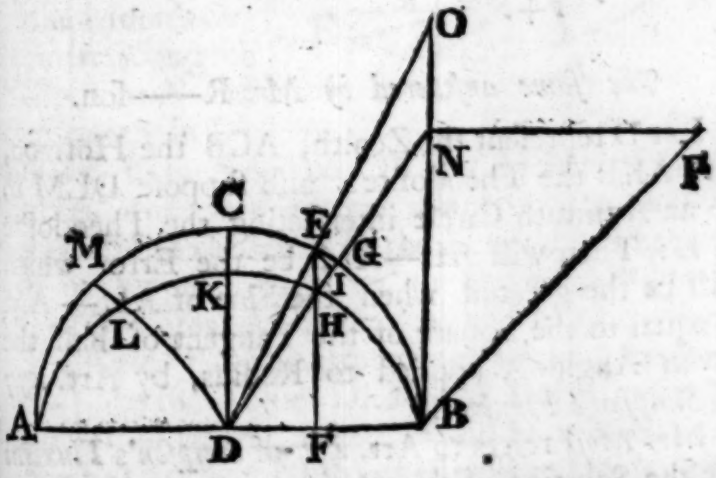
PRBLEM LX. *Anſwered by Mr.* Widd.

Let ABCD be the Plane of the Horizon and BDG the true Angle thereon, which is equal to the Repreſentation of the Angle BDE the Angle meaſured on the Theodolite; draw the Tangent BO meeting DG and DE, produced, in N and O reſpectively: Then, DB being ſuppoſed equal to Unity, if NBP be the Inclination of the two Planes, and NP be drawn perpendicular to BO, BP will be = BO. Therefore putting the Coſine of the Inclination = *c*, and BN = *x*, we ſhall have

have $c : x :: 1$ (Rad.) : $\frac{x}{c} = BP\ (=BO)$; whence the Tangent of the Angle EDG will be expreſſed by $\frac{x - cx}{c + x^2}$ which, by the Problem, is a Maximum; conſequently its Fluxion $c\dot{x} - c^2\dot{x} - x^2\dot{x} + cx^2\dot{x} = 0$, and $x = \sqrt{c}$: Which Value ſubſtituted in $\frac{x - cx}{c + x^2}$ gives $\frac{1 - c}{2\sqrt{c}}$ for the Tangent of the Error ſought.

The ſame anſwered by Mr. E. B—M. (*r*)

Let ACBD be the Plane of the Horizon and AKBD the Orthographick Projection of the Plane



of the Theodolite thereon; alſo let EDB be the true Angle, when the Inſtrument is parallel to the Horizon, and GDB the Orthographick Projection of an equal Angle, in the Plane of the Theodolite; draw DC and EF perpendicular to AB, and EI perpendicular to DG; then, it is evident, from the Principles of Orthographick Projection in No. 1, that DG, EF and the Semi-Ellipſis

AKB will interſect each other in the ſame Point H, and that the Angle EDG will expreſs the Error. Therefore, s, c and v denoting the Sine, Coſine and Verſed Sine of the Inclination, alſo y and x the Sine and Coſine of the Arch EB, we ſhall have $FH = cy$, $DH = \sqrt{1 - s^2y^2}$, and $EH = vy$: Moreover, by ſimilar Triangles, DH : DF :: EH : $EI = \frac{vy\sqrt{1-y^2}}{\sqrt{1-s^2y^2}}$ the Sine of the Error, which by the Queſtion is to be a Maximum; therefore $\frac{v^2y^2 - v^2y^4}{1 - s^2y^2}$ a Maximum; which put in Fluxions and reduced gives $y = \frac{\sqrt{v}}{s}$, whence the Error itſelf may be had, its Sine being expreſſed by $\frac{1-c}{1+c}$.

The ſame anſwered by Mr. R——ſon.

Let D repreſent the Zenith, ACB the Horizon, and AKB the Theodolite; alſo ſuppoſe DLM to be an Azimuth Circle interſecting the Theodolite in L: Then will AL—AM be the Error which will be the greateſt when the Sine of AL—AM is equal to the Square of the Tangent of half the given Angle A applied to Radius, by Art. 477 of *Simpſon*'s *Fluxions*.

Mr. *Moſs* refers to Art. 477 of *Simpſon*'s *Fluxions* for the Solution of this Problem.

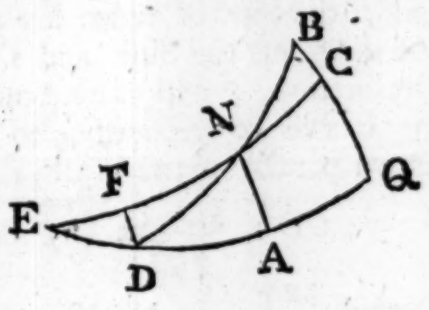
If the greateſt Error be required, when the Inclination of the two Planes is equal to 10 Degrees, we ſhall, by the firſt Method, have $x\ (=\sqrt{c}) = .9923748$ the Nat. Tang. of 44° 46′ 51″, and $\frac{1-c}{2\sqrt{c}} = .0076544$ the Nat. Tang. of 26′ 18″ the Error ſought.

By

By the ſecond Method $s = .1736482$, $v = .0151922$, $y = .7098066$ the Sine of 45° 13′ 9″, $x = .7043968$, the Sine of 44° 46′ 51″, and $\frac{1-c}{1+c} = .0076542$ the Sine of 26′ 18″ the required Error. And

By the third Method .0874887 is the Tangent of half A, and .0076542 is the Sine of 26′ 18″ the Error required.

PROBLEM LXI. *Anſwered by the Propoſer* Curioſus.



Let EQ be an Arch of the Equator, EC an Arch of the Ecliptic, and DNB an Arch of the Moon's Orbit interſecting the Ecliptic in N and the Equator in D; alſo let BQ, NA and DF be Arches of Great Circles, the two former perpendicular to the Equator and the latter to the Ecliptic: Then, if DB be ſuppoſed equal to 90 Degrees, BQ will expreſs the greateſt Declination, and EQ or, which is the ſame, ED will expreſs the Right Aſcenſion. But, ſince the Angles DEN and DNE are conſtant, it is evident that ED will be a Maximum when FD is a Maximum; that is when DN is equal to 90 Degrees, or when B and N coincide: For, *Sine* DEN : *Sine* DNE : : *Sine* DN : *Sine* ED, which, it is plain, will be a Maximum when the Sine of DN is a Maximum, or Radius.

Hence the Angle DEN being equal to 23° 29′, and the Angle DNE = 5°, we ſhall have in the Triangle DEF, the Angle E and the Perpendicular DF (= the Angle N) given; whence the Hypothenuſe DE will be found to be 12° 38′ 1″ 44‴.

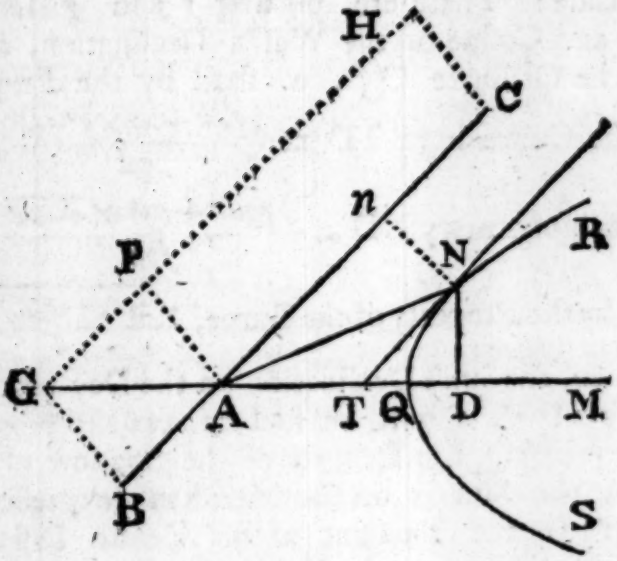
PROBLEM LXII. *Anſwered by Mr.* E. B—M.

It is evident, from Art. 467 of *Simpſon*'s *Fluxions*, that the Path of the Shadow of any Point in the upper Edge of the Wall will be an Hyperbola; therefore, if the Tranſverſe (t) thereof be denoted by Unity, its Conjugate will be expreſſed by $\frac{2p\sqrt{gb}}{f}$, where p denotes the Sine Complement of the Sun's Declination, b the Sine of his Meridional Altitude, g the Sine of his Depreſſion, and f the Sine of twice his Declination. Let D and d expreſs the Sine and Coſine of the Wall's Declination, x and y the Abſciſſa and Ordinate of the Curve correſponding to the Tangent thereof drawn parallel to the Wall; then by Trigonometry $d : y :: D : \frac{Dy}{d} =$ (by the Property of the Curve) $\frac{2x+2xx}{1+2x}$; but if $c = \frac{2p\sqrt{gb}}{f}$, y will be $= c\sqrt{x+x^2}$, and therefore $\frac{Dy}{d} = Dc\sqrt{x+x^2} = \frac{2x+2x^2}{1+2x}$; whence, putting $A = \sqrt{\frac{1}{d^2-c^2D^2}}$, we ſhall have $x = \frac{Ad-1}{2}$, and conſequently $y = \frac{c}{2}\sqrt{A^2d^2-1}$; which being known, the Sun's Azimuth may be found, and from thence the Time required.

The

The ſame anſwered by the Propoſer Curioſus.

Let BC be the Baſe of the Wall, and BG (or AP) its perpendicular Height (repreſented by Unity), and let SQR be the Path deſcribed, on the



Plane of the Horizon, by the Shadow of any Point P in the Edge, or Angle GH of the Wall; moreover, let NT, parallel to BC, be a Tangent to the Curve in N, interſecting the Axis (or Meridian) AM in T; let N*n* be perpendicular to BC, and join A, N.

It is evident, becauſe N is the neareſt Point in the Curve to the Baſe BC, that the Shadow of the Wall will be the ſhorteſt poſſible, when that of P falls upon N, or when the Azimuth of the Sun is expounded by the Angle MAN; in which Circumſtance, the Shadow of the Line GH falling upon the Tangent NT, the perpendicular Diſtance thereof from the Wall is truly repreſented by N*n*.

Now

Now, if the Sine of the Sun's Declination be denoted by d, its Coſine by p, and the Coſine of the Latitude by c, $\frac{pd}{c^2-d^2}$, and $\frac{p}{\sqrt{c^2-d^2}}$ will expreſs the Semi-tranſverſe and Semi-conjugate of the Hyperbola, which is the Curve deſcribed in this Caſe: Therefore, putting f and g for the Sine and Coſine of the Wall's Declination, and x for the Ordinate QD, we ſhall by the Property of the Curve have $TD = \frac{2pdx+x^2\times\overline{c^2-d^2}}{pd}$, and by Trigonometry $ND = \frac{2pgdx+gx^2\times\overline{c^2-d^2}}{fpd} =$ alſo, by the Property of the Curve, $\frac{\sqrt{2pdx+x^2\times c^2-d^2}}{d}$, whence, the Sun's Declination on the Day propoſed being 22° 5′, x will be had $= .226248$; which added to AQ the Length of the Shadow of AP when the Sun is on the Meridian (expreſſed by .3401361 the Tangent of his Zenith Diſtance) the Sum will give the Value of AD $= .566384$; from whence, that of DN (.3470224) being alſo known, the Angle DAN expreſſing the Sun's Azimuth will be given ($= 31° 29′ 44″$) and from thence the Hour of the Day $= 9^h\ 45^m\ 27^s\ 28^t$, or $2^h\ 14^m\ 32^s\ 32^t$. *Q. E. I.*

Mr. *R——ſon* anſwered this Problem after the ſame manner as above.

PROBLEM

PROBLEM LXIII. *Anſwered by* Proteus.

Let $a+z=x$, and $a+u=y$, then $2\dot{z}=2\dot{x}$ and $3\dot{u}=3\dot{y}$; conſequently the given Equation becomes $\frac{2\dot{z}}{a+z}+\frac{3\dot{u}}{a+u}=\frac{\overline{a+z}|^3\dot{u}}{a\times\overline{a+u}|^3}$, and therefore $2a\dot{z}\times\overline{a+u}|^3+3a\dot{u}\times\overline{a+u}|^2\times\overline{a+z}-\overline{a+z}|^4\times\dot{u}=0$. Put $Au^n=z$, and let the two Values of z and $\dot{z}$ be ſubſtituted in the Expreſſion preceding, then by the Reſolution of Fluxional Equations we ſhall have $z=-u-\frac{u^2}{4a}+\frac{71u^3}{24a^2}-\frac{1003u^4}{192a^3}+\frac{6273u^5}{640a^4}$ &c. $=a-y-\frac{\overline{y-a}|^2}{4a}+\frac{71\times\overline{y-a}|^3}{24a^2}-\frac{1003\times\overline{y-a}|^4}{192a^3}+\frac{6273\times\overline{y-a}|^5}{640a^4}$ &c. and therefore $x=2a-y-\frac{\overline{y-a}|^2}{4a}+\frac{71\times\overline{y-a}|^3}{24a^2}-\frac{1003\times\overline{y-a}|^4}{192a^3}+\frac{6273\times\overline{y-a}|^5}{640a^4}$ &c.

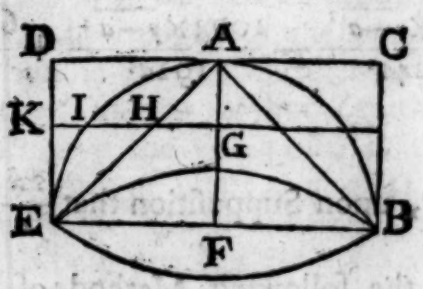
Mr. *Trott*, upon Suppoſition that $\frac{2\dot{x}}{x}+\frac{3\dot{y}}{y}=\frac{x^1\dot{x}}{ay^3}$, gives the following Method of Solution.

Let $\frac{p\dot{x}}{x}+\frac{r\dot{y}}{y}=\frac{x^m\dot{x}}{ay^n}$ be the general Equation expreſſing the Relation of y and x; then if, according to Art. 262, *Simpſon*'s *Fluxions*, each Side thereof be multiplied by x^py^r we ſhall have $px^{p-1}\dot{x}\times y^r+x^p\times ry^{r-1}\dot{y}=\frac{x^{m+p}\dot{x}}{ay^{n-r}}$; in which the former Part of the Equation is known to denote

denote the Fluxion of $x^p y^r$, and the latter, when $\dot{x}=r$, the Fluxion of $\frac{x^{m+p+1}}{m+p+1\times a}$; therefore the Relation of the Fluents in the Case proposed will be expressed by $x^2y^3 = \frac{x^6}{6a} + D$; which, when x and y each equal to a, becomes $a^5 = \frac{a^5}{6} + D$; therefore $D = \frac{5a^5}{6}$, and the Fluent sought $x^2y^3 = \frac{x^6+5a^6}{6a}$.

PROBLEM LXIV: *Answered by Mr.* E. B—M.

Let AF $=b$, FG $=u$, AG $=b-u$, EF $=d$, HG, IG or KG $=y$, t the Time in which a Cylinder, the Diameter of whose Base is EB and Altitude



AF, will discharge a Quantity of Liquor, equal to its Content, with the initial or greatest Velocity, and the Time sought $=z$; then $\dot{z} = -\frac{ty^2\dot{u}}{d^2\sqrt{bu}}$. Now, when the Vertices are upwards, the Abscissa x is $=b-u$, but when inverted $=u$; therefore, from the Property of the Curve, find the Value of y^2, in Terms of u and given Quantities, which substitute in the above Equation, and the Fluent properly corrected will give the Value of z, the Time required.

In

In the Cylinder, where $y^2=d^2$, $\dot{z}$ is $=\frac{-t\dot{u}}{\sqrt{hu}}$, and $z=2t$.

In the Hemiſphere, where $y^2=b^2-u^2$ and $b=d$, $\dot{z}=\frac{tu^2\dot{u}-tb^2\dot{u}}{b^2\sqrt{bu}}$, and therefore $z=2t\times\overline{1-\frac{1}{5}}=2t\times.8$. And,

In the Cone, where $y^2=\overline{b-u}^2$, $\dot{z}=\frac{-t\dot{u}\times\overline{b-u}^2}{b^2\sqrt{bu}}$; and $z=2t\times\overline{1-\frac{3}{5}+\frac{1}{5}}=2t\times.53$.

Therefore the Ratio required is as 15, 12 and 8 reſpectively.

The ſame anſwered by the Propoſer ΦΙΛΟ Σ.

Let a denote the Semidiameter of the Baſe and alſo the Altitude (AF) of each Veſſel, and let the variable Altitude (FG) of the Liquor be denoted by x. Then the Fluxions of the Solidity of the three Figures, at that Altitude, will be expreſſed by $4p\dot{x}a^2$, $4p\dot{x}\times\overline{a^2-x^2}$, and $4p\dot{x}\times\overline{a-x}^2$, and the correſponding Fluxions of the Times will be as $\frac{4p\dot{x}a^2}{\sqrt{x}}$, $\frac{4p\dot{x}\times\overline{a^2-x^2}}{\sqrt{x}}$, and $\frac{4p\dot{x}\times\overline{a-x}^2}{\sqrt{x}}$, or as $4pa^2x^{-\frac{1}{2}}\dot{x}$, $4p\times\overline{a^2x^{-\frac{1}{2}}\dot{x}-x^{\frac{3}{2}}\dot{x}}$ and $4p\times\overline{a^2x^{-\frac{1}{2}}\dot{x}-2ax^{\frac{1}{2}}\dot{x}+x^{\frac{3}{2}}\dot{x}}$ reſpectively. Whence the Fluents of theſe, or $4p\times2a^2x^{\frac{1}{2}}$, $4p\times\overline{2a^2x^{\frac{1}{2}}-\frac{2x^{\frac{5}{2}}}{5}}$ and $4p\times\overline{2a^2x^{\frac{1}{2}}-\frac{4ax^{\frac{3}{2}}}{3}+\frac{2x^{\frac{5}{2}}}{5}}$ muſt therefore be as the Times themſelves: Which (when $x=a$) are to each other in the Ratio of 2, $2-\frac{2}{3}$, and $2-\frac{4}{3}+\frac{2}{5}$, or of 15, 12, and 8 reſpectively. *Q. E. I.*

Note, If the Veſſels be inverted, or ſet with their Vertices downwards, and then ſuffered to empty themſelves, the Ratio of the Times will be found as 15, 7, and 3 reſpectively.

Meſſ. *R——ſon*, *Moſs*, and *Ghis*, alſo, anſwered this Problem.

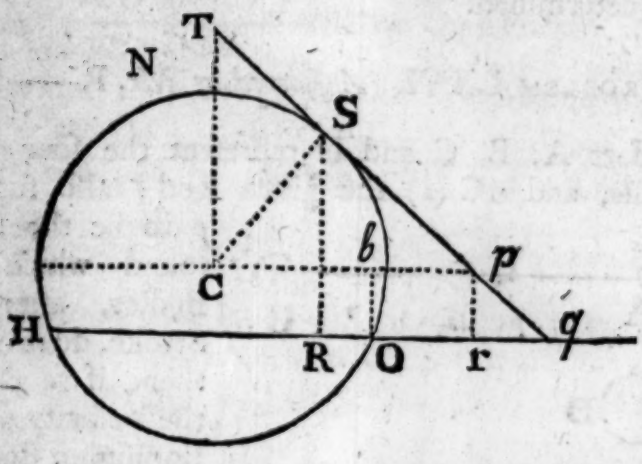
PROBLEM LXV. *Anſwered by Mr.* R——ſon.

Suppoſe the Degree of Heat to be as the mth Power of the Sun's Altitude into the nth Power of the Time of his Continuance above the Horizon, and let a and b be the Sine and Coſine of the given Latitude, alſo c and d the Sine and Coſine of the given Declination to the Radius 1; put S = the Semi-diurnal Arc, A = the Arc from Noon to the Time when this Phenomenon happens, and x and y for the Sine and Coſine thereof: Then $bd+acy$ will expreſs the Sine of the Sun's Altitude; and, it being certain that the greateſt Heat cannot happen before Noon, $\overline{bd+acy}|^m \times \overline{S+A}|^n$ muſt be a Maximum, and its Fluxion $macy\dot{y} \times \overline{S+A} + n\dot{A} \times \overline{bd+acy} = 0$. But by the Property of the Circle, $\frac{-\dot{y}}{x}$ is $= \dot{A}$, and conſequently $\overline{S+A} \times x - \frac{ny}{m} = \frac{n}{m} \times \frac{bd}{ac}$; from whence, by the Method of Trial and Error, or by infinite Series, the Value of x or y may be determined.

The ſame anſwered by the Propoſer Mr. Toſt.

Let the Circle HNSO be the Parallel of the Sun's Declination on the propoſed Day, and let HO be the common Section of the Plane thereof with the Plane of the Horizon; alſo let NH be the Semi-diurnal Arc of the Parallel, and NS the

the Arc thereof correſponding to the Time from Noon when the Phenomenon happens: Let the

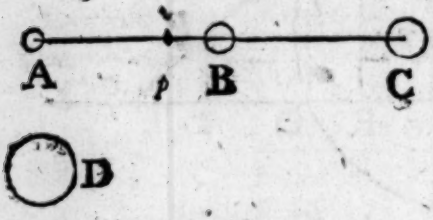


Tangent TS*q* be drawn and thro' the Center C draw C*p* parallel to HO; alſo draw SR, O*b* and *pr* perpendicular to HO; then, if the Arc HN be denoted by *a*, the perpendicular *pq* by *d*, the Arc HS by *z*, the Line SR by *y*, and the Tangent TS by *x*, we ſhall, as SR is in a conſtant Ratio to the Sine of the Sun's Altitude, have $z^m \times y^n$ a Maximum, therefore its Fluxion $mz^{m-1}\dot{z} \times y^n + ny^{n-1}\dot{y} \times z^m = 0$: Whence $m\dot{z}y = -n\dot{y}z$, and $\frac{m}{n} \times \frac{\dot{z}y}{-\dot{y}} = z$. But as S*q* is denoted by $\frac{\dot{z}y}{-\dot{y}}$, it appears that, in the required Circumſtance, $m : n ::$ the Arc HS : Tangent S*q*; therefore, as the Arch NS is expreſſed by $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9}$, &c. and S*q* by $\frac{1 + d\sqrt{1+x^2}}{x}$; we ſhall have $m : n :: a + x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7}$, &c. :

$\frac{1+d\sqrt{1+x^2}}{x}$; from whence the Value of x may be determined.

PROBLEM LXVI. *Answered by Mr.* R——son.

Let A, B, C and D represent the four given Balls, and AC (l) the given Rod; also suppose p to be the Point round which the Bodies, after the Stroke, do revolve; then, if c denote the Velocity of the impinging Body D and x the Distance of the Body A from p, we shall have $D+A : A :: cD : \frac{cAD}{D+A}$ the Momentum of A after the Impact, therefore its Velocity will be $\frac{cD}{D+A}$. But the Velocities of Bodies being as their Distances from the Center of Motion, we shall have $x : \frac{1}{2}l-x :: \frac{cD}{D+A} : \frac{\frac{1}{2}l-x \times cD}{x \times D+A}$ the Velocity of B, and $x : l-x :: \frac{cD}{D+A} : \frac{l-x \times cD}{x \times D+A}$ the Velocity of C; whence the Momentum of B will be $= \frac{\frac{1}{2}l-x \times cBD}{x \times D+A}$, and that of C $= \frac{l-x \times cCD}{x \times D+A}$. Moreover $x : \frac{1}{2}l-x :: \frac{\frac{1}{2}l-x \times cBD}{x \times D+A} : \frac{cBD \times \frac{1}{2}l-x^2}{x^2 \times D+A}$ the Part of B's Momentum



Momentum which acts at A, or the Motion loſt by D in generating the Momentum of B; in like Manner $\frac{cCD \times \overline{l-x}^2}{x^2 \times D+A}$ will be that Part of D's Momentum loſt by generating C's Momentum. Whence $\frac{cAD}{A+D} + \frac{cBD \times \overline{\frac{1}{2}l-x}}{x \times A+D} + \frac{cCD \times \overline{l-x}}{x \times A+D}$ $= \frac{cAD}{A+D} + \frac{cBD \times \overline{\frac{1}{2}l-x}^2}{x^2 \times D+A} + \frac{cCD \times \overline{l-x}^2}{x^2 \times D+A}$ the whole Motion loſt in D by the Impulſe; conſequently $x = \frac{Ax^2 + B \times \overline{\frac{1}{2}l-x}^2 + C \times \overline{l-x}^2}{Ax + B \times \overline{\frac{1}{2}l-x} + C \times \overline{l-x}}$.

Coroll. 1. Hence, it is manifeſt that the Point *p* thus determined, is the Center of Percuſſion or Oſcillation, and will remain invariable let the Magnitude of the Ball D and the Velocity with which it impinges be what they will.

Coroll. 2. It is alſo evident, from hence, that if the Number of Balls A, B, C, *&c.* be ſuppoſed infinite and equal; or, which is the ſame Thing, if a Cylinder or Parallelopipedon, *&c.* be thus ſtruck at one of their Extremes, the Point round which they will revolve will be the Center of Percuſſion or Oſcillation. *See Page 209 at the end*

The ſame anſwered by the Propoſer Waltonienſis.

After the Impulſe D will continue to move uniformly in the ſame Direction as before; its Velocity will be equal to the abſolute Velocity of A, immediately after the Stroke; and the Center of Gravity of the three Balls faſtned to the Rod will move uniformly in a Direction parallel to that in which D moves, the Ball at the ſame Time revolving uniformly about that Center.

The

The Velocity of any one of thoſe three Balls about their Center of Gravity, I ſhall call its circulatory Velocity; and its Weight into that Velocity, its circulatory Motion.

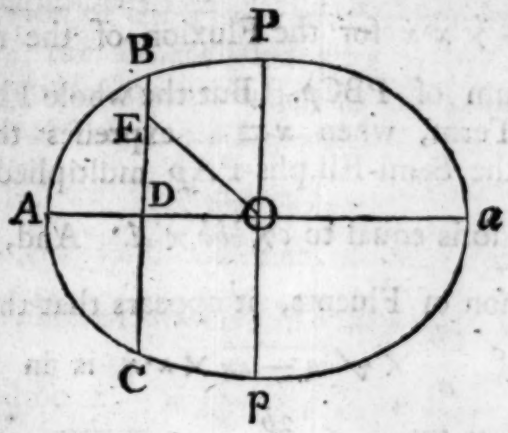
Let v be the circulatory Velocity of A; then its Weight being 1, v will be its circulatory Motion: And, the circulatory Velocities of A, B, and C being as their Diſtances from their Center of Gravity (which Diſtances are as 4, 1, and 2 reſpectively) $\left.\begin{matrix}\frac{1}{4}v\\ \frac{1}{2}v\end{matrix}\right\}$ will be the circulatory Velocity of $\left\{\begin{matrix}B\\C\end{matrix}\right\}$ and $\left.\begin{matrix}\frac{1}{4}v\\ \frac{1}{2}v\end{matrix}\right\}$ its circulatory Motion.

To generate $\left\{\begin{matrix}v\\ \frac{1}{2}v\\ \frac{3}{4}v\end{matrix}\right\}$ circulatory Motion in $\left\{\begin{matrix}A\\B\\C\end{matrix}\right\}$ D muſt loſe $\left\{\begin{matrix}v\\ \frac{1}{2}v\\ \frac{3}{4}v\end{matrix}\right\}$ Motion, by the Property of the Lever. Therefore $\frac{15}{8}v$ is the whole Quantity of Motion deſtroy'd in D. Now that being equal to the Quantity of Motion generated in the other three; if u be put for the Velocity of the Center of Gravity of thoſe Three, $u+2u+3u$ muſt be $=\frac{15}{8}v$; therefore $u=\frac{5}{16}v$. Moreover $v+u$ will be D's Velocity after the Impulſe, and $4v+4u$ its Quantity of Motion. But before the Stroke its Quantity of Motion was $4c$, therefore $4c-4v-4u$ is the Motion D loſes by the Colliſion; conſequently $4c-4v-4u=6u$, $5u=\frac{25}{16}v=2c-2v$, $v=\frac{32}{57}c$, $u=\frac{10}{57}c$, and $v+u=\frac{14}{19}c$. Theſe being known, and the Length of the Rod being given, the Places of the Balls may, at any Time, be eaſily found.

PROBLEM

PROBLEM LXVII. *Anſwered by the Propoſer.*

Let AP*ap* be any Section of the given Spheroid perpendicular to the Axis of Motion; alſo let



BDC be an Ordinate to the Tranſverſe Axis A*a*; and, from (E) any Point therein, to the Center of the Ellipſis, let EO be drawn: Then, if the given Angular Celerity, at the Diſtance 1, be denoted by *c*, the Celerity of a Particle at E will be expounded by $c \times OE$; and its Force to turn the Ellipſis about its Center, by $c \times OE \times OE$. Which laſt Expreſſion, if OD be put $= d$ and $DE = z$, will become $c \times dd + zz$: And ſo the Fluent of $c \times \overline{dd + zz} \times \dot{z}$, which is $cz \times \overline{dd + \frac{1}{3}zz}$, will be the Momentum of the whole Line DE: The Double whereof $(c \times BC \times OD^2 + \frac{1}{3}BD^2)$ when E coincides with B, will conſequently be the true Momentum, or Force, of the whole Ordinate BC.

Let OD, now conſidered as variable, be denoted by x; calling AO, a; and its Semiconjugate OP, b: Then, BD being $= \frac{b}{ba}\sqrt{aa - xx}$, our Expreſſion for the Momentum of BC will become

come $c \times \frac{2b}{a}\sqrt{aa - xx} \times \frac{bb}{3} + \frac{aa - \frac{1}{3}bb}{aa} \times x^2$:

Which, drawn into $(\dot{x})$ the Fluxion of OD, gives $c \times \frac{2b}{a}\sqrt{aa - xx} \times \frac{1}{3}bb\dot{x} + c \times \frac{2b}{a}\sqrt{aa - xx} \times \frac{aa - \frac{1}{3}bb}{aa} \times x^2\dot{x}$ for the Fluxion of the required Momentum of PBCp. But the whole Fluent of the firſt Term, when $x = a$, expreſſes the Area (A) of the Semi-Ellipſis PAp multiplied by $c \times \frac{bb}{3}$; and ſo is equal to $c \times \frac{1}{3}bb \times A$. And, by the Compariſon of Fluents, it appears that the whole Fluent of $\frac{2b}{a} \times \sqrt{aa - xx} \times x^2\dot{x}$ is in Proportion to (A) That of $\frac{2b}{a} \times \sqrt{aa - xx} \times \dot{x}$, as $\frac{1}{4}aa$: 1 ; and therefore is equal to $\frac{1}{4}a^2A$. Whence, the Fluent of our ſecond Term being $\frac{1}{4}aaA \times c \times \frac{aa - \frac{1}{3}bb}{aa}$ $(= cA \times \sqrt{\frac{1}{4}aa - \frac{1}{12}bb})$ the Fluent of the whole Expreſſion will, conſequently, be $cA \times \sqrt{\frac{bb}{3} + \frac{aa}{4} - \frac{bb}{12}} = \frac{1}{4}cA \times \sqrt{aa + bb}$: The Double of which or $\frac{1}{4}c \times \sqrt{\overline{AO}^2 + \overline{PO}^2} \times$ *Area of the Ellipſis*, is evidently the true Momentum of the Ellipſis: Which, if p be put for the Area of a Circle whoſe Radius is Unity and the Ratio of a to b be denoted by that of 1 to r, will become $\frac{1}{4}cpr \times \sqrt{1 + rr} \times a^4$.

Let now the Diſtance of this Section APap from the Center of the Speroid be denoted by y, and ſuppoſe the Semitranſverſe Axis of the greateſt Section, paſſing thro' the Center, of this Spheroid

to

to be denoted by g; then, by the Property of the Circle, we have $a^2+y^2=g^2$ and consequently $a^2=g^2-y^2$ and the Momentum of the Section above found $=\frac{1}{4}cpr\times\overline{1+rr}\times\overline{gg-yy}^2$, which multiplied by $\dot{y}$ gives $\frac{1}{4}cpr\times\overline{1+rr}\times\overline{g^4-2g^2y^2+y^4}\times\dot{y}$ for the Fluxion of the Force of the Spheroid itself: Whereof the Fluent when $y=g$ is $\frac{1}{4}cpr\times\overline{1+rr}\times\frac{8}{15}g^5$; the Double of which is the required Momentum of the whole Spheroid.

COROLLARY.

Since the solid Content of the Spheroid is $\frac{4prg^3}{3}$, it follows that the Momentum of the Spheroid will be $=\frac{c}{5}\times\overline{gg+rrgg}\times Solidity$; where $gg+rrgg$ expresses the Sum of the Squares of the two Semi-Axes of the Section of the Meridian.

With Regard to the Remarks on Fluents by *Waltonienſis*, Mr. *W. L.* obſerves; "That the "Writers on Fluxions refer to the Nature of the "Problem, from whence the Fluxionary Equation "is derived, for the Correction of the Fluent, as "the only General Method" And that; "their "not having pointed out (which would be very "hard to do) thoſe particular Forms of Expreſſi-"ons that will admit of no conſtant Quantity in "the Fluent, beſides thoſe included in the given "Equation, ought not therefore to be conſtrued "as a Defect, much leſs a Miſtake, ſince no Au-"thor" (that he knows of) "has ſuppoſed that "it is always poſſible for one of the ſimple variable "Quantities to have its Beginning, when the other "has attained any given Value."

He, however, does Juſtice to the above-named Gentleman's Invention, and expreſſes himſelf well pleaſed with his Manner of proceeding on the two Examples produced by him.

See p. 102.

It is really ſomewhat remarkable, that there ſhould be a Kind of Fluxionary Equations, capable of being ſolved intirely by the *Direct Method*, independent of any ſort of Aſſumption, either, of variable or conſtant Quantities. Of this Kind are the two Equations above ſpecified (ſee our laſt Number, p. 102). For, with Regard to the former of Them, $\frac{a\dot{x}^2}{\dot{y}} = y\dot{x} - x\dot{y} + b\dot{y}$, it is evident, by taking the Fluxion (ſuppoſing $\dot{y}$ conſtant) that $\frac{2a\dot{x}\ddot{x}}{\dot{y}} = y\ddot{x}$, (as *Waltonienſis* gives it). When, the whole being diviſible by $\ddot{x}$, there is no Need of having Recourſe to the *Inverſe Method*, nor of aſſuming any conſtant Quantity at all; ſince, by ſubſtituting $\frac{2a\dot{x}}{\dot{y}}$, for its Equal $\ddot{y}$, in the given Equation, the fluxional Quantities will intirely vaniſh, and we ſhall then have $4ax = yy + 4ab$ for the (only) true Relation of the Fluents (as *Waltonienſis* juſtly obſerves). And, by the very ſame Method the Relation of the Fluents, in his ſecond Example may be found. And it is alſo evident that all Fluxionary Equations whatever, whoſe Fluxions (taken as above) are diviſible by the ſecond Fluxion, are determinable the ſame way; and will admit of no conſtant Quantity beſides Thoſe included in the original Equation.

The Method of ſolving theſe Kinds of Equations was firſt of all pointed out (if I miſtake not) by Mr. *Simpſon* in the *Weekly Oracle*; and afterwards at p. 169 of his *Treatiſe of Fluxions*. But as to his Correction of the Fluent, it ſeems to have

have been from the Nature of the Problem, and not from the Equation; this laſt Part of the Invention belonging to *Waltonienſis*.

To the Author of the Mathematical Exerciſes.

SIR,

AS a Lover of Juſtice and ingenious Men, I cannot, without Indignation, ſee the Reputation, due to the Invention of Perſons of ſuperiour Genius, beſtowed on Thoſe who have, *baſely*, copied from them, whilſt the original Authors are not ſo much as named. This appears to be the Caſe with regard to the Forms, or Tables of Fluents, ſo often quoted in all our periodical Mathematical Productions your own not excepted); there being, ſcarce, a ſingle Fluent in any of our preſent Mathematical Author's * but what may be met with in Sir *Iſaac Newton*'s *Quadratures*, and in *Cotes*'s *Harmonia Menſurarum*.

I here ſend you a new Species of Fluents, not to be found in Mr. *Emerſon*'s Table, nor even in the Works of thoſe two Great Men.

I am, Sir, &c.

HAVERFORDIENSIS.

* We preſume that our Correſpendent has not ſeen Mr. *Simpſon*'s laſt Book of Fluxions.

Forms.	Fluxions.	Fluents.
1	$\frac{\overline{a-bz^n}^{-\frac{1}{2}} dz^{\frac{1}{2}n-1}\dot{z}}{k+lz^n}$	$\frac{dA}{n\sqrt{bk^2+alk}}$
2	$\frac{\overline{a-bz^n}^{-\frac{1}{2}} dz^{\frac{1}{2}n-1}\dot{z}}{\overline{k+lz^n}^2}$	$\frac{dA\times\overline{bk+\frac{1}{2}al}}{n\times\overline{bk^2+alk}^{\frac{3}{2}}}$
3	$\frac{\overline{a-bz^n}^{-\frac{1}{2}} dz^{\frac{1}{2}n-1}\dot{z}}{\overline{k+lz^n}^3}$	$\frac{dA\times b^2k^2+bkal+\frac{3}{8}a^2l^2}{n\times\overline{bk^2+alk}^{\frac{5}{2}}}$
4	$\frac{\overline{a-bz^n}^{\frac{1}{2}}\cdot dz^{\frac{1}{2}n-1}\dot{z}}{k+lz^n}$	$\frac{dA\sqrt{bk+al}}{nlk^{\frac{1}{2}}} - \frac{db^{\frac{1}{2}}A}{ln}$
5	$\frac{\overline{a-bz^n}^{\frac{1}{2}} dz^{\frac{1}{2}n-1}\dot{z}}{\overline{k+lz^n}^2}$	$\frac{daA}{2nk^{\frac{3}{2}}\sqrt{bk+al}}$
6	$\frac{\overline{a-bz^n}^{\frac{1}{2}} dz^{\frac{1}{2}n-1}\dot{z}}{\overline{k+lz^n}^3}$	$\frac{daA\times\overline{4bk+3al}}{8nk^{\frac{5}{2}}\times\overline{bk+al}^{\frac{1}{2}}}$

Note, in theſe Forms which give the whole Fluent of the propoſed Fluxion (when the Quantity, $a-bz^n$, under the Radical Sign becomes equal to nothing) A expreſſes the Semiperiphery of a Circle whoſe Radius is Unity; a, b, n and k any affirmative Numbers at Pleaſure; and l any poſitive Number, or a Negative One provided the ~~Value of~~ Value of $bk+al$ is poſitive.

A Collection *of* Problems *to be answered in the next Number.*

Problem LXVIII. *By Mr.* Thomas Marſhall *of* Blenchland.

THERE are three Numbers whoſe Sum is 12, and the Sum of the firſt and ſecond is to that of the ſecond and third as 3 to 5; alſo the Difference of the two laſt is to their Sum as 1 to 5: What are the Numbers?

Problem LXIX. *By the ſame.*

A Grazier, having laid out 200 *l.* in Oxen at 7*l.* a Head and Sheep at 12 *s.* a Head, ſold them again, the Oxen at 8 *l.* a Head and the Sheep at 13*s.* and gained 25*l.* by the Bargain: How many of each had he?

Problem LXX. *By* Lepus.

To determine the Values of x and y in the following Equations, *viz.* $xx+yy=aa$, and $\overline{x-y}^2 + b\times\overline{x+y} = c^2$.

Problem LXXI. *By Mr.* Moſs.

The Hypothenuſe of a right-angled Triangle, and the Rectangle under one of the Legs and their Sum, being given; to determine the Triangle.

Problem

Problem LXXII. *By* Lupus.

Having given the Length of a Line, drawn from the acute Angle at the Baſe to the Middle of the Perpendicular, of a right-angled Triangle; as alſo the Length of a Perpendicular drawn from the Right Angle to the Hypothenuſe; 'tis required, from thence, to determine the Triangle.

Problem LXXIII. *By* Proteus.

A Line drawn from either of the acute Angles of a right-angled Triangle, to the Center of the inſcribed Circle, is a Mean-Proportional between the Hypothenuſe and the Exceſs of the Hypothenuſe above the Leg oppoſite to the propoſed Angle. A Demonſtration of this is required.

Problem LXXIV. *By* ΣΟΦΟΣ.

If thro' the Focus of a given Ellipſis any Right Line be drawn, terminating in the Periphery, the Rectangle under the Parts thereof intercepted by the Focus and the Periphery, applied to the whole Line, will be a conſtant Quantity. Required the Demonſtration.

Problem LXXV. *By* ΣΕΜΝΟΣ.

Having the Angle which any Diameter of a given Ellipſis makes with the greater Axis; to find the Angle which its Conjugate Diameter makes with the ſame Axis, together with the Lengths of Both.

Problem LXXVI. *By* Waltonienſis.

A having three Guineas in his Hand and *B* two, agree to toſs up each his own Pieces, and that he who brings moſt Heads uppermoſt ſhall take all five:

five: And they likewiſe agree, that if both bring the ſame Number of Heads, to toſs up again (and ſo continue) 'till it ſhall appear who is the Winner. Quere which of them has the Advantage? and how much it is?

PROBLEM LXXVII. *By Mr.* Moſs.

The Inclination of a Plane to the Horizon being given; to determine the Length thereof, ſo that a Ball, in deſcending, from the Top, along the ſaid Plane by the Force of its own Gravity, ſhall be a given Interval of Time longer in reaching the Horizon than if let fall freely from the Top of the ſaid Plane.

PROBLEM LXXVIII. *By* Curioſus.

To find at what Degree of Elevation a Ball muſt be projected, ſo that the Area of the Curve deſcribed in its Flight may be the greateſt poſſible.

PROBLEM LXXIX. *By* Proteus.

On a particular Day of the Year, in the Lat. of 51° 30′ N. I obſerved the Sun to riſe on a certain Point of the Compaſs; 11 Hours after which I perceived my Shadow to be projected towards the very ſame Point. What was the Sun's Declination at that Time?

PROBLEM LXXX. *By* ΦΙΛΟΞΕΝΟΣ.

The Poſition of the Sun and Moon, with Reſpect to the Earth, being given; to determine the Poſition of the higheſt Part of the Ocean, ariſing from their mutual Gravitation.

PROBLEM LXXXI. *By* Waltonienſis.

Three Balls A, B, and C, of equal Weight, being faſtned to a String ſtretched ſtraight out;

A at one End, B in the Middle, and C at the other End; ſuppoſe B to be impelled, with a Celerity *c*, in a Direction at Right-angles to the String: It is required to find the Paths the other two Balls will take, and when they will come together, all Three being ſuppoſed to move on a horizontal Plane; and the Length of the String being $2a$.

A ſhort Examination of a pretended Anſwer of the late *Compiler of the* Ladies Diary *to the Charge of Falſhood and Ignorance exhibited againſt him, in our Third Number: With ſome Obſervations on his laſt ſcurrilous Pamphlet.*

IT will not, I hope be imagined by Any One, that I am contending for the *Laſt Word*, much leſs, for Superiority, with this Opponent: As I engaged in the Defence of an Author, for whom I have the higheſt Eſteem, whoſe Character has been long traduced, by him, with Impunity, I find myſelf once more obliged to take up the Cudgels.

Whoever has had Curioſity enough to look into this Controverſy, muſt have obſerved that it chiefly turns upon the three following Points.

1. The Author's Definition of a Fluxion;

2. His Method of ſolving a Fluxionary Equation;

3. The Aſſertion of this Opponent, that *the Author publiſhes Nothing but by the Aſſiſtance of Others.*

In

In Relation to the firſt of theſe Particulars, it muſt appear clear, I think, to Every One, from our Obſervations in N° III. that the Definition in Queſtion is entirely free from thoſe monſtrous Abſurdities which his Critic has endeavoured to fix upon it. All that he has been able to prove is, that *it* is not exactly the ſame with *that* of Sir *Iſaac Newton*. This Mr. *Simpſon* himſelf tells us in his Preface; and ſhews alſo wherein the Difference conſiſts; and what induced him to think that a ſmall Deviation from that Great Man might be of Advantage. If any Fault can poſſibly be found with the Author here, it is for uſing the Word *Fluxion* in a Senſe ſomewhat different from *that* put upon it by moſt other Writers. However, as he has defined his Meaning, and not given his Method for *that* of Sir *Iſaac Newton*, I cannot ſee how this deſerves Cenſure: Had he aſſumed any other *Name*, he might have been, juſtly, condemned as guilty of an unpardonable piece of Vanity; the Difference between his Method and *that* of the ſaid illuſtrious Author being too inconſiderable to authorize any ſuch Step. Beſides it is well known that we have other Books amongſt us, *under the ſame Name*, that are built upon Principles differing much more from *thoſe* of Sir *Iſaac Newton*.

As to the ſecond of the three Heads above ſpecified, I muſt deſire my Reader to look back to *Numb.* II. p. 37, and to *Numb.* III. p. 93 and 94; where may be ſeen the Subſtance of what has been offered, on both Sides, in Relation to this controverted Equation. At the End of a general Demonſtration given in p. 94, confirming the Truth of the Author's Obſervation will be found the following Words, in relation to the ſaid Demonſtration.

" If my Antagoniſt can diſcover any Thing " falſe or equivocal in This, I here promiſe, pub- " lickly, to acknowledge my Miſtake, and aſk " his Pardon: But, in the mean time, muſt take " the Liberty to put him in mind that, if, after " all his extraordinary Confidence and Oſtentation, " he is not able to point out the leaſt Flaw in it " (which no Man living can) his Readers will have " more Reaſon than ever to look upon him as " groſsly ignorant in the moſt common Parts of " the Subject."

It might be imagined that an Invitation like this muſt, infallibly, have brought this Part of the Diſpute, at leaſt, to an Iſſue, either by giving him an Opportunity to Triumph, or obliging him to Silence; I ſay, this might be imagined, but Thoſe that think thus have but an imperfect Idea of the *Fortitude* and *Perſeverance* of our Opponent. They will ſee him again, renewing the Attack, in a Manner that ſhews him to be invulnerable. I ſhall here tranſcribe what he has thought proper to advance, *farther*, in Anſwer to a Charge ſo particular and preſſing.

" It is Matter of Fact" (ſays he) " that there " are Millions of Problems wherein *x* and *y* don't " begin together; and are we therefore to drag an " Exponential Quantity into the Calculation to " make them do ſo? Certainly, this is only em- " barraſſing the Work. For when the Fluent " found will produce its proper Fluxionary Equa- " tion and all the Operations [*can be**] performed " as well WITHOUT as WITH the Exponential, or " better; then, certainly the EXPONENTIAL is need- " leſs. This is what you ought to have REPLIED to,

* I have taken the Liberty to add theſe two Words, becauſe the Senſe is imperfect without them.

" inſtead

" inſtead of which you bring a long Calculation " to ſhew the Depth of your Learning; which is " as much to your Purpoſe as if you had given " us the Calculation of the Moon in her Orbit."

Having given you *his* Arguments, at length, I ſhall now take the Liberty to make a few Remarks upon them.

" There are" (ſays he) " Millions of Problems " wherein x and y don't begin together;" this is granted; " are we therefore to drag an Expo- " nential Quantity into the Calculation to make " them do ſo?" No, certainly, if the Nature of the Problem does not require it. " This is em- " barraſſing the Work;" 'tis ſo, nay, 'tis worſe; 'tis even making the Work wrong if no ſuch Correction is required. " For when the Fluent " found will produce its proper Fluxionary Equa- " tion and all the Operations [*can be*] perform'd " as well WITHOUT as WITH the Exponential, or " better; then, certainly, the EXPONENTIAL IS " NEEDLESS." Very true! when all the Operations can be performed as well *without* as *with* the Exponential: But, the Misfortune is, theſe Operations can't be *at all* performed without the *Exponential*; except in one particular Caſe out of an infinite Number; as I have demonſtrated to all the World.

This is the very Point on which the Diſpute depends: The Queſtion is not, whether a Thing be *needleſs* when it *is needleſs* (for all that this egregious Trifler has ſaid amounts to no more) but whether the Exponential, which he *ignorantly* objected againſt, is *needleſs*. Never did little Pretender, pinch'd by the Weight of Argument, ſhuffle more, or make a more contemptable Figure than he does *here*. Is this convincing the Publick that Mr. *Simpſon* is ignorant in the Method of

Fluxions.—But, let us hear him further. "This "is what you ought" (continues he) "to have "replied to" (*i. e.* whether a Thing is unneceſſary when it is unneceſſary), "inſtead of which you "bring a long Calculation, to ſhew the Depth "of your Learning; which is as much to your "*Purpoſe*, as if you had given the Calculation of "the Moon in her Orbit."

So then, according to this Gentleman, a General Solution of a Problem (for ſuch is *That* referred to) is of no ſort of Uſe in determining the Relation and Values of the Quantities belonging to the Problem; and, though it has been univerſally proved that the Equation objected to, contrary to his ignorant Aſſertion, would be *intirely uſeleſs without the Exponential* *, except in one ſingle Caſe out of an Infinite Number; yet, all this, it ſeems, *is no more to the* Purpoſe *than calculating the Moon in her Orbit.*

* In a Poſtſcript to a Letter, dated *March* 8, 1752, which I received from *Waltonienſis*, is the following Remark relating to this Equation. "What a Dunce was *Heath* to object to Mr. *Simpſon*'s Correction of this Fluent $y = -c \times \overline{xx + 2ax + 2aa} + d\dot{M}^{\frac{x}{a}}$! I took no Notice of it 'till lately. A Gooſe of a Critic indeed! He rightly calls himſelf *Critic Anſer*. Mr. *Simpſon*'s Obſervation on that Fluent is what no Writer had made before. Mr. *Emerſon* is defective in that Particular, and at Page 51 of his Fluxions leaves his Fluent $z = a + x +$ Number of the Log. $\frac{x}{2.30258a}$; ſo that it ſuits but one Caſe, and cannot be corrected for any other without looking back and correcting the Fluent of $\dot{x} = \frac{a\dot{\psi}}{\psi}$; which, tho' it was not agreeable to his Method of Writing to take Notice of it in that Place, he ought to have adverted to where he ſpeaks of the Correction of the Fluents."

If

If this requires an Anſwer, I confeſs I am at a Loſs for One!—I ſhall proceed now to the third Particular of the Controverſy, relating to his Calumny that, " Mr. *Simpſon* publiſhes Nothing " but by the Aſſiſtance of others: " And, to place this Matter in the cleareſt View, ſhall firſt tranſcribe (from N° III. p. 94 and 95) the Invitation given to this *bold* Aſſertor to make good a Charge, in which the Character of an Author is ſo deeply ſtruck at; whoſe Name and Works are known to the greateſt Mathematicians in *Europe*.

The Quotation is as follows:

" There is alſo another Thing if he" (this Antagoniſt) " has not given up all Pretenſions to " Truth and Morality, that I beg to remind him " of; which is, that he would make it appear, as " he ſays *it is known that Mr.* Simpſon *publiſhes* " *Nothing but by the Aſſiſtance of Others*, who thoſe " OTHERS are to whom Mr. *Simpſon* is under ſuch " SINGULAR Obligations—I myſelf have had the " Pleaſure of knowing Mr. *Simpſon* for ſome Years, " and dare affirm in the Face of the World that, " if ever Man alive was wronged by an Impu- " tation of this Kind, Mr. *Simpſon* is."

" If the HONEST Gentleman, who ſays *it is* " *known that the Compiler* (meaning Mr. *Simpſon*) " *publiſhes Nothing but by the Aſſiſtance of Others*, " can make it appear who thoſe Aſſiſtants are, or " ſhew that the Author's Copy has been reviſed or " even ſo much as inſpected by any other Perſon " whatever (except in Relation to his firſt Perfor- " mance, where he was under a Neceſſity of ſub- " mitting to it) I ſhall make no Scruple to own " that I have formed a very wrong Judgment of " the Perſon whoſe Cauſe I have engaged in. On " the other Hand, if he is not able to do it (as " I am confident he is not) I leave it to be de-

" termined

" termined by Mankind whether the Heart or the " Head of the Man is the worſer."

This is the Manner in which he was called upon to make good his Aſſertion: We ſhall now ſee what He has thought proper to advance as an Anſwer.

" Tho' our POLITE LADY" (meaning the Author of the MATHEMATICAL Exerciſes) " draws " her Arguments from a Fund of odorous Perfumes, we don't chuſe to deal in, and denies us " the Ability to write OUR OWN Productions, yet " ſhe takes it AMISS being told the Author of " Mr. *Simpſon*'s Fluxions publiſhes nothing but " by Aſſiſtance of others. But pray, MADAM, " why are you ſo captious about TRIFLES? when " we all know who corrected the PROOF SHEETS " of your favourite Fluxions, when and where. " And would any Author of Judgment truſt the " Correction of Proof-Sheets, and Reputation in " difficult Science, to an AUXILIARY; eſpecially " living himſelf within THE REACH OF THE " PENNY POST from the Printer?

" How then is your favourite Author WRONGED " of Mathematical Reputation, by our ſaying he " was aſſiſted? Don't we all know he ſtands ſupported by AMANUENSES, REVISERS, and ADVOCATES (yourſelf one) to which may be " added GRAMMATICAL PEDANTS, Haberdaſhers " of POINTS and Syllables; Refiners of Diction; " Correctors of Sentiment; CASTRATORS of " redundant Members of PLAGIARISM; Collectors " from FRENCH and LATIN mathematical Authors; and Puffers of Performances!"

Is this ſuch an Anſwer as might be expected from *any* Man, not *intirely* inſenſible to all Regards of Truth, Character and Decorum? " Don't we all " know," (ſays this wicked Prevaricator) " that " the

« the Author ſtands ſupported by *Amanuenſes*, « *Reviſers, &c. &c.* »

I anſwer, before the whole World, that neither He, nor no Man living, knows it; and that every Thing he has here ſaid, or inſinuated, is *utterly* falſe, except with Regard to the Correction of the Proof Sheets. Theſe were indeed ſent to me, the firſt time, to correct, to the Copy (unleſs the Author himſelf happened to be in Town) by which Means the Printer had them again two Days ſooner than if they had been ſent to the Author's Place of Abode in the Country. But the ſecond Proof of every Sheet was always ſent to the Author himſelf, whether in Town or Country: Nor was there a Word, or a Point, altered from the Original Copy, but by the Author himſelf. This I think myſelf, in Juſtice, obliged to declare: Nor could I, if I would, be ſilent on this Occaſion, without appearing as *ignominious* as this *invidious* Calumniator *himſelf*: Since it is well known to all the Author's Friends (who are neither few nor inconſiderable) how little he ſtands in need of Helps of *that Sort*; whatever may have been his Unhappineſs in Point of Education.

Having now gone thro' all the principal Parts of the Controverſy; I ſhall here add a few Obſervations on our Adverſary's laſt Productions, the *Lady's Philoſopher*, and the new *Palladium*.

In the former of theſe Pamphlets He has condeſcended to favour the Publick with our Method (at N° IV. p. 99) for *approximating the Value of a Decimal Fraction*: Which he has included in three Pages. He has done the ſame with Reſpect to the *Memorial Verſes for the new Style* (*vid.* p. 134.) Theſe he has given, metamorphized and diſguiſed in ſix Pages. Laſtly, He has thought proper alſo to allow a Place to the Method laid down by Mr.

Simpſon,

Simpſon, at p. 47 of his Eſſays, for finding the Equation of a Planet's Orbit; with which he makes a Shift to fill up three Pages more: So that upon the whole, there are twelve Pages in this Collection wherein we and our Friends have the Honour to *ſhine*, not including, more than double the Number, in which we have the Honour to be abuſed.

As we have always looked upon the Publick to be the proper Judges in Matters of this Nature, we ſhould have paſſed theſe *Obligations* over in Silence (as we have already done Others) had it not been for a ſingle Remark of three Lines; Which muſt not eſcape with Impunity.

This Remark, or Note, is at the End of an Example, taken from Mr. *Simpſon*'s Eſſays above-mentioned; wherein (after having given the Operation at length by Logarithms (which the Author had omitted) and found the Reſult to be 66° 51′ 52″) he has theſe Words:

"Not 66° 52′ 50″, or 51″, being 58 or 59 "Seconds too much, as Mr. *Simpſon* has deter- "mined it, to whom we recommend the Practice "of Truth."

Here, it ſeems, the Author ſtands charged with an Error of 58 or 59 Seconds. Now, ſuppoſing that ſuch an Error *had* crept into his Calculations, I don't ſee that this ſhews a Diſregard to Truth, nor how it would be ſufficient to juſtify ſuch an *inſulting* Manner of Expreſſion. What, then muſt we think when this pretended Miſtake is found to be nothing more than a Preſs Error, by putting the Minutes in the Place of the Seconds; and that it is taken Notice of and corrected (as it actually is) by the Author himſelf in his *Errata*.—Judge from hence, what the Man is, whom we have to deal with!

I muſt

I muſt now take ſome Notice of a new Opponent, who appears againſt us under the Banners of our Enemy, in a very hoſtile Manner.

He ſeems greatly irritated at a Remark, at p. 138, in our N° IV. relating to a Problem propoſed by him. As I ſhall, at all Times, be ready to do Juſtice, I ſhall make no ſort of Scruple to acknowledge here, that I miſconceived his Meaning when I wrote the ſaid Remark, and to allow alſo that the Problem, *as it is now printed*, is plain enough to be underſtood and perfectly *determinable*. But, then the Propoſer ought to have recollected that the Manner of Expreſſion, under which it was ſent to us, *was not the ſame*. Nor can I, *yet*, ſee the Propriety of Talking of a Thing as *reſting* on a Plane, when it is ſuppoſed to be *in Motion* along the Plane.

However, be this as it will, the *extraordinary* Epithets he uſes on this Occaſion might in my humble Opinion have very well been ſpared without hurting his Cauſe. Neither can I ſee how He can reconcile his own Conduct, to the Rules of ſtrict Juſtice, in falling upon an Author in ſo abrupt a Manner under a Pretence of defending a Perſon who never was (that I know of) moleſted by him. It will appear to the World, a bold Aſſertion that, "Mr. *Simpſon* ſeems IGNORANT of "the Method of correcting Fluents." A Perſon that can ſay thus much, ſhould, if he has a proper Regard to his own Character, be well aſſured that he has the Truth on his Side. But this, I apprehend, will not be found to be the Caſe with our angry Antagoniſt: Nor will the Example he produces (in which he ſeems more ſolicitous to ſhew his Abilities than to make good his Charge) appear at all to his Advantage. This Example is Prob. XV. in Sect. XI. of the Author's *Doctrine and Application* (p 516)

plication of Fluxions. The Subject there propoſed is to find the Equation of the *Curve of Purſuit*, deſcribed by a Body in giving direct Chace to another Body moving uniformly in a Right Line.

In order to bring out the Concluſion as neat and ſimple as poſſible, the Author makes Uſe of ſuch Lines and *Data* as appear moſt conducive to that End; and therefore begins his Solution by ſuppoſing a Tangent to be drawn to the Curve to *meet* the Abſciſſa at Right Angles: In Conſequence whereof, when he comes to correct the Fluent, he makes ($\dot{x}$) the Fluxion of the Abſciſſa equal to Nothing. Which every one, the leaſt acquainted with the Buſineſs of Tangents, knows cannot, poſſibly, be otherwiſe, as $\dot{y}$ is conſtant, and the Tangent perpendicular to the Axis by Conſtruction. Nevertheleſs this angry Gentleman, roundly, affirms that "$\dot{x}$ may be either equal to nothing, "or to a finite Quantity;" and upon no better Authority aſſerts that "the Author ſeems ignorant in the Method of correcting Fluents."

He may poſſibly alledge, in his own Defence, that if the Computation was to begin at any other Point of the Curve, $\dot{x}$ would *then* be a finite Quantity. If ſo, I cannot but congratulate him on the *Fertility* of his Invention: But in the mean time ſhould be glad to know what Right he has to make a Suppoſition contrary to the Author's, and then condemn him upon that, very, Suppoſition.

Is this a Mark of Candour and Genius?

The ſame Spirit appears in his affirming that "the Author had ſolved only one particular Caſe "of the Problem." This Doughty Antagoniſt ought to have remembered that the Problem was *To find the Equation of the Curve*; which Equation *is found*, in the neateſt Manner, ſo as to give the true Figure of the Curve and the Relation of its

Abſciſſa

Abſciſſa and Ordinate, in all poſſible Caſes whatever. Nay, the very Equation which he makes ſuch an oſtentatious SHEW with, is deducible therefrom, in a Manner far more ſimple and plain than That by which he gives it, as is evident from the Solution to Prob. XLVII. in our third Number: Againſt which he has indeed thought proper to object; but it had been more modeſt in him to have been ſilent *here*, eſpecially as his own Concluſion ($z = \frac{r^2\sqrt{n^2 + a^2} + rn}{1 - rr}$), *to which he gives the Preference*, happens to be intirely *falſe*.

— Thus much for this *minute* Critic and the *Lady's Philoſopher*; I come now to the *Palladium*: Wherein (at p. 61 and 62) are the following, *remarkable*, Paragraphs; Which I have tranſcribed, without the Alteration, or Omiſſion, of a ſingle Point or Syllable.

1° The *Contributors* and Public are deſired to excuſe the *Errors* and *Abſurdities* in this Year's *Lady's Diary*, we not being allow'd the Correction of the Proof-Sheets, for dark Reaſons: But they will find their Loſſes amply repaired in our *Lady's Philoſopher* and future *Palladiums*. See our Lady's Philoſopher for the reſt.

2°. Correſpondents to the *Lady's Philoſopher* are deſired to ſend their Letters, (*Poſt paid*) at every Opportunity, directed for the *Triumvirate*, at Mr. *Price*'s, in the *Poultry*, near the *Manſion-Houſe*, *London*, whither, and to no other Place, Correſpondents of the *Palladium*, or any other Work of ours, are deſired to ſend their Letters to prevent Abuſe, and enable us to oblige them, who are deſired to diſregard all other Advertiſements (without our Conſent) requiring the contrary.

Gentlemen, *Ladies*, and *others*, willing to ſubſcribe to the *Lady's Philoſopher*, are deſired to ſend their *Names* and Places of Abode to the Place aforeſaid, that they may be printed in our Catalogue, as Promoters to ſo uſeful a Work; deſigned to comprehend a Variety of curious and intereſting Subjects, for general Improvement: And alſo what there was not Room for in the *Lady's Diary*, and this *Palladium*. *The Proprietor of the* Lady's Diary *being the* Relict *of the late Mr.* Hen. Beighton, F. R. S. *who we are ready to ſerve, the Contributors ſending her Letters to the Place aforeſaid, and not to* Stationers Hall, *where our* Palladium *Letters have been detain'd, ſhe ſhall be ſure to have them unopened: Being firm in the Widow's Cauſe, while the* Mercenary *ſeek to oppreſs.*

I ſhall not here take upon me to paſs Judgment on theſe two *dark* Paragraphs; but content myſelf with offering a few Queries thereon, to the Conſideration of the Publick.

1°. Whether *his deſiring the Publick to excuſe the Errors and Abſurdities in this Year's Lady's Diary*, BECAUSE NOT CORRECTED BY HIMSELF, has not ſomething in it highly inconſiſtent and ridiculous.—Either, he muſt mean that, he was not *allowed* to have any hand at all in the ſaid Diary, or that he was not *allowed* to have the Correction of *his own* Proof-Sheets.

If the former be the Caſe, how can he take upon him to paſs Sentence againſt what he had not ſeen? If the latter, what muſt the World think of his Abilities as a Compiler, in ſending *ſuch Errors and Abſurdities* to the Preſs?

2°. Whether, *the dark Reaſons*, hinted at by this Writer, have not been placed in a very clear

Light

Light by our Correſpondent *Honeſtus* (*vid. Gazetteer*, *Dec.* 4 and 13, 1751; and N° II.)

3°. Whether, his decrying the *Lady's Diary* and ſolliciting a Subſcription for the *Lady's Philoſopher*, does not look as if he had a Deſign to eſtabliſh the *One* upon the *Ruins* of the *Other*.

4°. Whether, this Manner of proceeding looks like *being firm in the Widow's Cauſe*, who is the Proprietor of the ſaid Diary? and whether his advertiſing in order to get all the Letters deſign'd for the Diary into his own Hands (without publickly avowing that he had the Widow's, or the Company of Stationers, Authority for ſo doing) can admit of any favourable Interpretation?

Though we have not Room HERE *to animadvert on the Impropriety and Badneſs of ſeveral of the Queſtions and Solutions in this Palladium; yet the ſecond of theſe Sollutions muſt not be paſſed over: The Principal there laid down* (*viz.* that the Sides of a Triangle are reciprocally as the biſecting Lines) *is ſo notoriouſly repugnant to the moſt obvious Propoſitions in Geometry, that the Compiler's publiſhing it to the World muſt either* ARGUE AN UNCOMMON DEGREE OF IGNORANCE, *or, what is worſe, the moſt cruel Ill-Nature in him, to ſuffer a Perſon to appear in ſo bad a Light, who has not only been a ſteady Contributor, but has alſo furniſh'd him with a Number of pretty Problems.*

—We are oblig'd, again, to poſtpone the Publication of the remaining Part of *Honeſtus*'s Letter to a further Opportunity.

J. TURNER.

See the end of the Book the last N°

THE

Mathematical Magazine

AND

Philosophical Repository

CONTAINING A VARIETY OF

ORIGINAL PIECES

IN ALL PARTS OF

MATHEMATICAL SCIENCE,

&c. &c.

VOL. I.

This is the last number of the exercises continued from Page 183 of the forepart of this Book.

MATHEMATICAL EXERCISES:

CONTAINING,

I. The Concluſion of the Principles of Dialling.

II. Anſwers to the Problems propoſed in the fifth Number.

By *JOHN TURNER.*

N°. VI.

WREXHAM:

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Sold alſo by J. F. and C. RIVINGTON, St. Paul's Church-Yard, LONDON, where the preceding Numbers may be had; in which are contained the Orthographic and Stereographic Projections of the SPHERE; the Inveſtigation of the Sums of SERIES; the Principles of DIALLING; A Method to find a VULGAR FRACTION nearly equal in Value to any DECIMAL One; ſome Obſervations on FLUENTS, MEMORIAL VERSES adapted to the New Style; together with a Number of curious PROBLEMS, in various Branches in the Mathematics, and their ANSWERS.

[Price ONE SHILLING.]

184

MATHEMATICAL

EXERCISES:

CONTAINING,

I. The Conclusion of the Principles of Dialling.

II. A Reflexion to the Problems proposed in the Fifth Number.

By JOHN TURNER.

No. VI.

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Sold also by J. F. and C. Rivington, St. Paul's Church-Yard, London, where the preceding Numbers may be had; in which are contained the Orthographic and Stereographic Projections of the Sphere; the Investigation of the Sums or Series; the Principles of Practical Measuring; a Method to find a Vulgar Fraction nearly equal in Value to any Decimal One; some Observations on Decimals; Memorial Verses adapted to the New Style; together with a Number of curious Problems, in various Branches in the Mathematics, and their Answers.

[Price One Shilling.]

ANSWERS *to the* PROBLEMS *propoſed in the* FIFTH NUMBER.

PROBLEM LXVIII. *Anſwered by Mr.* Henry Watſon.

LET x, y and z denote the three required Numbers, then per Queſtion $x+y+z=12$, $x+y:y+z::3:5$ and $z-y:z+y::1:5$; whence $y=12-x-z$, $\frac{3z-5x}{2}$ and $\frac{2z}{3}$ reſpectively, $z=\frac{36-3x}{5}$ and $3x$ reſpectively, and $x=2$ conſequently $z=6$, and $y=4$.

The ſame anſwered by Tommy Tangent.

Let $a=12$, $x=$half the Sum of the ſecond and third Numbers, and $z=$ half their Difference; then $x+z=$the third Number, and $a-x-z=$the Sum of the firſt and ſecond; whence *by the Problem* $3:5::a-x-z:2x$, therefore $5a-5x-5z=6x$, or $5z=5a-11x$: alſo $1:5::2z:2x$, whence $5z=x=$ (alſo) $5a-11x$; therefore $x=\frac{5a}{12}=5$, and $z=1$; conſequently 2, 4 and 6 are the Numbers required.

This Problem was alſo anſwered by Meſſ. *Noorthouck*, *Weſton*, *Trott*, *Phips*, *Widd*, *Barker*, and *E. B—M.*

PROBLEM LXIX. *Anſwered by Mr.* Barker, *of* Weſthall, Suffolk.

Let x denote the Number of Oxen, and y the Number of Sheep; then if a repreſent the Shillings in £200, b the Shillings in £7, c 12 Shillings, d the Shillings in £8, m 13 Shillings, and p the Shillings in £25. we ſhall have $bx+cy=a$, and $dx+my=a+p$; whence, in the firſt, $x=\frac{a-cy}{b}$, and in the laſt $\frac{a+p-my}{d}$; which compared together gives $y=\frac{da-ba-bp}{cd-bm}=100$ the Number of Sheep required, therefore $x=20$ the Number of Oxen.

The ſame anſwered by Neptune *of* Sunderland.

Let x repreſent the Number of Oxen, and y the Number of Sheep, then $7x+.6y=200$, and $8x+.65y=225$; whence $x+.05y=25$, and $7x+.35y=175$ which ſubtracted from the firſt Equation gives $.25y=25$; or $y=100$; conſequently $x=20$.

This Problem was, alſo, anſwered by Meſſ. *Noorthouck*, *Weſton*, *Trott*, *Phips*, *Widd*, *Watſon*, *Tommy Tangent*, and *E. B.—M.*

PROBLEM LXX. *Anſwered by Mr.* Widd, *of* Whitby.

Suppoſe $x-y=u$, and $x+y=z$; then $u^2+z^2=2a^2$ and $u^2+bz=c^2$, whence $2a^2-z^2+bz=c^2$; which reduced gives $z=\frac{b}{2}+\sqrt{2a^2-c^2+\frac{b^2}{4}}$ from whence u, x and y may be found.

Tommy Tangent anſwered this Problem nearly in the ſame Manner.

The ſame anſwered by Mr. Trott.

Let the laſt Equation be taken from twice the firſt, and we ſhall have $\overline{x+y}|^2 - b \times \overline{x+y} = 2a^2 - c^2$; from which, by compleating the Square $x+y$ will be found $= \frac{b + \sqrt{b^2+8a^2-4c^2}}{2}$; this ſubſtituted for $x+y$ in the ſecond Equation and the Root extracted gives

$$x-y = \sqrt{c^2 - \frac{b^2 + b\sqrt{b^2+8a^2-4c^2}}{2}}$$

which added to $x+y$ gives $x =$

$$\frac{b + \sqrt{b^2+8a^2-4c^2} + 2\sqrt{c^2 - \frac{b^2 + b\sqrt{b^2+8a^2-4c^2}}{2}}}{4};$$

but if $x-y$ be taken from $x+y$, y will alſo be found.

Mr. R—ſon's *Anſwer was much in the ſame Manner as Mr.* Trott's, *and*

Mr. Watſon *anſwered it as follows.*

By ſubſtituting $z+n=x$, and $z-n=y$, the given Equations will become $2z^2+2n^2=a^2$, and $4n^2+2bz=c^2$, therefore $4z^2-2bz=2a^2-c^2$; whence $z = \frac{b \pm \sqrt{b^2+2a^2-c^2}}{2}$ from which x and y may be found.

Mr Ghis's *Solution was nearly as Mr.* Watſon's; *and the Problem was, alſo, anſwered by Mr.* Noorthouck, E. B.—M, *Mr.* Barker *and* Neptune.

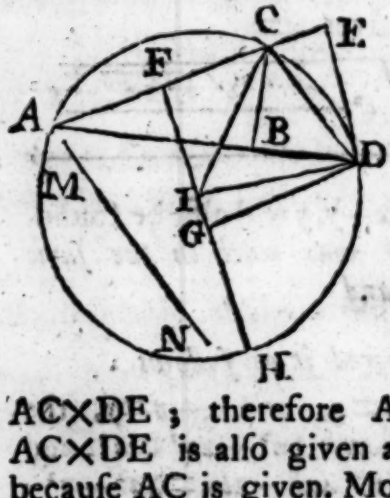
PROBLEM LXXI. *Anſwered by Mr.* Watſon.

Let the given Hypothenuſe be repreſented by a, the given Rectangle by b, one of the required Legs

Legs by x and the other by xy; then will $x^2y^2+x^2=a^2$ and $x^2y+x^2=b$, the former of which Equations divided by the latter gives $\frac{y^2+1}{y+1}=\frac{a^2}{b}$, whence $y=\frac{\sqrt{a^4+4ba^2-4b^2}+a^2}{2b}$.

This Problem was alſo anſwered algebraically by *Tommy Tangent*, Meſſ. *Phips*, *Weſton*, *Noorthouck*, *Widd*, *Barker*, *E. B.*—*M.* and *Neptune*.

The ſame anſwered Geometrically by Lupus.



Let ABC be the required Triangle, whoſe Hypothenuſe is AC, and let AB be produced to D ſo that BD may be equal to BC; alſo perpendicular to AC, produced, draw D E. It is evident that AD×BC is equal to AC×DE; therefore AD×BC being given, AC×DE is alſo given and conſequently DE, becauſe AC is given. Moreover, ſince BC=BD and the Angle CBD (CBA) a Right-Angle, it is evident that the Angle ADC is equal to Half a Right-Angle; from whence we have the following

CONSTRUCTION.

Upon the given Hypothenuſe AC let a Segment of a Circle be deſcribed to contain an Angle equal to Half a Right-Angle; biſect AC in F with the Perpendicular FH meeting the Periphery

phery of the Circle in H, and having made MN equal to the Side of a Square, whoſe Area is equal to the given Rectangle, take FG a Third Proportional to AC and MN; then draw GD parallel to AC and meeting the Circle in D; alſo draw AD and DC and make the Angle BCD equal to ADC, and the Thing is done.

Method of Calculation.

To I, the Center of the Circle, let CI and DI be drawn; then we ſhall have FC: IG (FG—FC) : : *Sine of* 45° : *Coſine of the* Angle DIG, which is the Difference of the Angles DCA and DAC. Theſe being now found, every Thing more required may be eaſily had.

Mr. R—ſon *ſent a Geometrical Conſtruction nearly ſimilar to the above,* and *Mr.* Ghis *ſent a very ingenious Conſtruction.*

PROBLEM LXXII. *Anſwered by Mr.* Noorthouck.

Let the biſecting Line be expreſſed by b, and the given Perpendicular by c, alſo let x expreſs Half the biſected Perpendicular and y the Segment of the Hypothenuſe contiguous to the acute Angle at the Baſe; then $y^2+c^2=b^2-x^2$, and by ſimilar Triangles $2x : c : : \sqrt{y^2+c^2} : y$, whence $2xy=c\sqrt{y^2+c^2}$, whence therefore x^2 (by the firſt Equation) $=b^2-c^2-y^2$, and by the Second $=\frac{c^2y^2+c^4}{4y^2}$, conſequently $y^4+\frac{5c^2-4b^2}{4}\times y^2 =-\frac{c^4}{4}$; which by ſubſtituting n for the Coefficient

ent of y^2 becomes $y^4 - ny^2 = -\frac{c^4}{4}$ whence

$$y = \sqrt{\frac{n + \sqrt{n^2 - c^4}}{2}}.$$

Neptune, Mr. Wid, E. B.—M, Tommy Tangent and Mr. Watson answered this Problem algebraically, as did also Mr. Phips by the Substitution of Sines and Cosines; and

Mr. Weston, *answered it as follows:*

By putting b for the bisecting Line, a for the given Perpendicular, x for the Segment of the Hypothenuse next the Vertical Angle, $2y$ for the bisected Perpendicular, and z for the Base; we have by similar Triangles $x : a :: a : \frac{a^2}{x} =$ the Segment of the Hypothenuse next the acute Angle at the Base; also $x^2 + a^2 = 4y^2$, $a^2 + \frac{a^4}{x^2} = z^2$, and $b^2 = z^2 + y^2$ (by 47. E 1.) From the Second of these Equations take the Third and we shall have $a^2 + \frac{a^4}{x^2} - b^2 = -y^2$; to the Quadruple of this last add the first, and we shall $5a^2 + \frac{4a^4}{x^2} - 4b^2 + x^2 = 0$, or $x^4 + \overline{5a^2 - 4b^2} \times x^2 = -4a^4$; which by writing $-2g$ for the Coefficient of x^2, and compleating the Square gives $x^4 - 2gx^2 + g^2 = g^2 - 4a^4$, therefore

$$x = \sqrt{:\sqrt{g^2 - 4a^4} + g}.$$

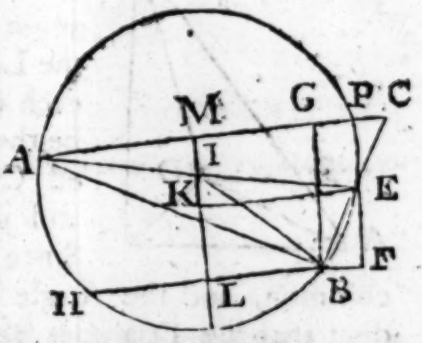
The same answered by Mr. Ghis.

Con-

CONSTRUCTION.

Upon the given biſecting Line AE, as a Diameter, let a Circle be deſcribed, in which let the Line ED be inſcribed equal to half the given Perpendicular; join A, D and produce DE to F ſo that EF may be equal to ED; draw FH parallel to AD and interſecting the Circle in B, alſo join A, B and thro' E draw BEC meeting AD produced in C and the Thing is done.

DEMONSTRATION.



Draw BG parallel to FD. Since the Angles CDE and EFB are each Right-Angles, the Angle DEC oppoſite the Angle FEB, and the Sides EF, ED equal, BE will be equal to EC. Moreover, by Reaſon of the Parallel Lines, AD, FH and DF, GB, GB will be equal to DF equal the given Perpendicular by Conſtruction; but the Angle ABC is a Right-Angle by Conſtruction, as alſo AGB (ADE), therefore ABC is the Triangle required. *Q. E. D.*

Method of Calculation.

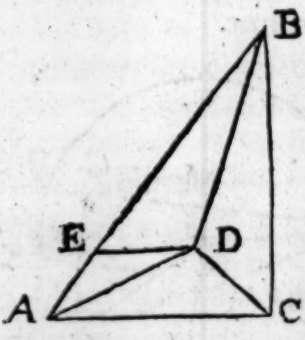
Biſect the Diameter AE with the Line MIL perpendicular to AC and FH, and draw EK parallel to AC; alſo join B, I.

In the Triangle KIE are given the Radius IE and the Line KI = ¼ BG (= ½ ED) whence the

Angles KIE, KEI and the Side KE are had. Then KI : I L : : (1 : 3 : :) *Sine* KEI : *Sine* LBI the Complement of the Difference of the Angles BEA and BAE; from whence the Angles themſelves, and what elſe is required, may be found.

Mr. R—ſon's *Anſwer was nearly in the ſame Manner as the above.*

PROBLEM LXXIII. *Anſwered by Mr.* R—ſon.

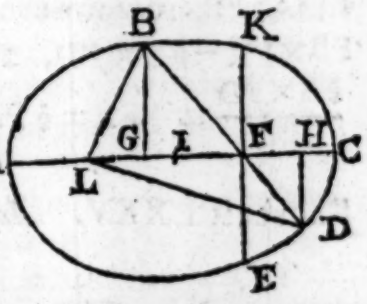


Let the Acute-Angles A, B of the Right-Angled Triangle ACB be biſected by the Lines DA, DB meeting each other in D, which will be the Center of the inſcribed Circle; make BE=BC and join D, E and D, C. Since the Side BE=BC, BD common, and the Angle DBE=DBC, it is evident that the Triangles BED, BCD will be equal and ſimilar in all Reſpects, and therefore the Angle BED=BCD=half a Right-Angle. But the Sum of the Angles BAD, ABD is=half a Right-Angle, conſequently their Supplement ADB equal to AED the Supplement of BED; therefore the Triangles AED, ADB ſimilar, whence AE : AD : : AD : AB. *Q. E. D.*

This was anſwered much in the ſame Manner, as above, by the Propoſer.

PROBLEM

PROBLEM LXXIV. *Anſwered by Mr.* R—ſon.



In the annexed given Ellipſe, whoſe Tranverſe is AC, Parameter KE, and Diſtance of the Foci LF, let BD be drawn thro' the Focus F. Join the Points L, B and L, D and let the Perpendiculars BG, DH upon the Tranverſe AC be drawn; then by the Property of the Ellipſe BL=AC—BF and DL=AC—FD; alſo, by Theo. 9, B. 2, Simpſon's Geometry, $GF=\frac{\overline{LF}^2-\overline{AC}^2+2AC\times BF}{2LF}$; whence by ſimilar Triangles BGF, DFH it will be BF: $FD::GF:\frac{FD\times\overline{LF}^2-\overline{AC}^2+2AC\times BF}{2LF\times BF}=FH.$

But the Property of the Ellipſe BF—KF : GF :: LF : AC, and FE—FD : FH :: LF : AC; whence by Equality BF—KF : KF (FE)—FD :: (FG : FH ::) FB : FD, and per Alternation and Compoſition of Ratio's 2BF : KF :: BF+FD (BD) : FD, and conſequently 2BF×FD=KF×BD, and thence $\frac{FB\times FD}{BD}=\frac{KF}{2}$, or ¼ the Parameter of the given Ellipſe. *Q. E. D.*

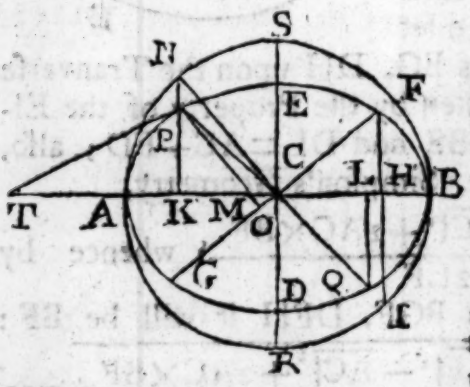
The ſame anſwered by the Propoſer, ΣΟΦΘΣ.

Thro' the Focus F of the given Ellipſe, whoſe Tranſverſe is AC, Parameter KFE, and Center I, draw the Right-line BFD, and perpendicular to AC draw BG and DH. Then by the Proper-

ty

ty of the Ellipse FI ; IA : : FB—FK : FG :: FK—FD : FH, and FB—FK : FK—FD : : FG : FH : : FB : FD ; whence FD×FB—FD×FK = FB×FK—FB×FD, 2FB×FD=FK×FB+FD, $\frac{FB \times FD}{BD} = \frac{1}{2}FK = \frac{1}{4}$ Parameter. *Q. E. D.*

PROBLEM LXXV. *Answered by Tommy Tangent.*



Let AEBD be the given Ellipse, FG the Diameter thereof which makes an Angle, with the greater Axis, equal to the given Angle; draw the Ordinate FH, perpendicular to the Transverse Axis AB, and let the said Ordinate be produced to meet the Periphery of a Circle, described on the Transverse Axis, in I; make CL=HI and draw the Ordinate LQ perpendicular to AB; lastly from the Point Q, thro' the Center, draw QCP and it will be the Conjugate Diameter required. (See Page 148 of the Mathematician.)

The same answered by Mr. R---son.

Let AEBD represent the given Ellipse, whose Transverse is AB and Conjugate ED; and let ASBR be the circumscribing Circle. Make the Angle ACP equal to the given Angle, and draw the Tangent PT (to the Ellipse) intersecting the Transverse produced in T, also let CF be drawn

drawn parallel to PT, then CP will be the Semi-diameter and CF the Semi-conjugate required. All which is evident from the Nature of the Ellipſe.

Method of Calculation.

Let PM be perpendicular upon PT, and thro' P draw NK perpendicular to AB; join C, N then KN : KP : : CA : CE : : *Cotangent* of the given Angle KCP : *Tangent* of the Angle KNC, from whence KN, KP, KC and the Semi-diameter CP all become known: Alſo $\overline{CA}|^2 : \overline{CE}|^2$ :: CK : KM (per Property of the Ellipſe), and KM : CK : : *Tangent* of the given Angle KCP : *Tangent* of the Angle PMK, whoſe Complement KTP=FCB is the Angle ſought. And laſtly the Semi-conjugate CF hence becomes known, it being $=\sqrt{\overline{CA}|^2+\overline{CE}|^2-\overline{CP}|^2}$ by the common Property of the Conjugate Diameters.

The ſame by the Propoſer ΣΕΜΝΟΣ.

Let AB be the greater Axis, ED the leſſer, PQ the Diameter given by Poſition, GF its Conjugate, ASBR a Circle deſcribed about the Ellipſe, NPK perpendicular to AB, and PMO perpendicular to the Curve and to the Diameter GF.

By common Properties, as the Parameter : AB : : (KM : KC : :) *Tangent* KCP : *Tangent* KMP, whoſe Complement MCO is the Angle ſought.

Again CE : CS (:: KP : KN) :: *Tangent* KCP : *Tangent* KCN ; and (by Trig.) *Sec.* KCN

KCN : *Sec.* KCP :: CN (CA) : CP ; whence CP is given : And in the very same Manne (the Angle BCF being found, CF will be given.

This Problem was answered Algebraically by Mr. Ghis.

PROBLEM LXXVI. *Answered by Mr.* R---son.

If A throws 1 Head and B none, the Probability of which is $\frac{3}{32}$, he then wins, or if he throws two Heads and B either none or but one, the Probability of which is $\frac{9}{32}$, he then wins likewise ; but if he throws three Heads, the Probability of which is $\frac{1}{8}$, he wins necessarily, and so the whole Probability of A's winning any particular Throw will be expressed by $\frac{3}{32}+\frac{9}{32}+\frac{1}{8}=\frac{8}{16}$; and in like Manner the Probability of B's winning will be found to be $\frac{1}{16}+\frac{1}{8}=\frac{3}{16}$, and consequently the Ratio of their Probabilities on any one Throw as 8 to 3 ; whence the Value of A's Expectation on the first Throw is 2l. 12s. 6d. and that of B's 19s. 8d$\frac{1}{4}$. ; and there will remain 1l. 12s. 9d$\frac{3}{4}$. to be contended for the second Throw, which being divided in the same Ratio as before, will give the Values of their Expectations on the second Throw : And by proceeding thus their Expectations on any succeeding Throws may be determined, they forming two Infinite Series, the corresponding Terms whereof are in the constant Ratio of 8 to 3, and

3, and conſequently their Sums in the ſame invariable Ratio. Whence, ſince the Value of both together is 5l. 5s. the particular Value of each will be 3l. 16s. 4d$\frac{1}{4}$. + $\frac{5}{11}$ and 1l. 8s. 7d$\frac{1}{2}$. + $\frac{6}{11}$ the whole of *A*'s and *B*'s Expectation reſpectively.

The ſame anſwered by the Propoſer, Waltonienſis.

At any particular Toſs, the Probability that *B* brings not one Head is $\frac{1}{4}$, and the Probability that *A* brings at leaſt one Head is $\frac{7}{8}$: Therefore the Probability that *B* brings not one Head and *A* wins the firſt Toſs is $\frac{1}{4} \times \frac{7}{8} = \frac{7}{32}$.

The Probability that *B*, at any particular Toſs, brings preciſely one Head is $\frac{1}{2}$, and the Probability that *A* brings either 2 or 3 Heads is $\frac{1}{2}$: Therefore the Probability that *B* brings preciſely one Head and *A* wins the firſt Toſs is $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

That *B*, at any Toſs, brings both Heads, the Probability is $\frac{1}{4}$; and that *A* brings all three Heads, the Probability is $\frac{1}{8}$: Therefore the Probability that *B* brings both Heads and *A* wins the firſt Toſs is $\frac{1}{4} \times \frac{1}{8} = \frac{1}{32}$.

Conſequently $\frac{7}{32} + \frac{1}{4} + \frac{1}{32} = \frac{1}{2}$ is the Probability that A wins the firſt Toſs.

But this is not all his Probability of winning, for he may bring as many Heads as *B* brings; and then he has the ſame Probability of winning he had at firſt; which Probability we will call *x*.

Now the Probability that, at any particular Toſs, neither *A* nor *B* brings a Head is $\frac{1}{32}$; that *A* and *B* bring each preciſely one Head is $\frac{3}{16}$; and that each brings two Heads preciſely is $\frac{3}{32}$. Therefore $\frac{1}{32} + \frac{3}{16} + \frac{3}{32} = \frac{5}{16}$ is the Probability

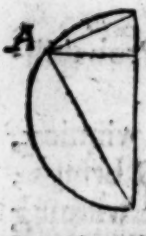
 that

that, at any particular Toss, both *A* and *B* bring the same Number of Heads: And consequently the Probability that *A* shall not win the first Toss, but sometime afterwards, is $\frac{5}{16}x$. Consequently $\frac{1}{2}+\frac{5}{16}x=x$, and $x=\frac{8}{11}$. Therefore 3l. 16s. $4\frac{4}{11}$d. is the Expectation of *A*, and 13s. $4\frac{4}{11}$d. his Advantage.

PROBLEM LXXVII. *Answered by* Micromegas.

Let *s* represent the Nat. Sine of the given Inclination, *a* the given Difference of the Time of descending along the Plane, and that of descending perpendicularly to the Horizon, and *y* the Time of Perpendicular Descent; then we shall have $a+y : y :: 1 \text{ (Rad.)} : s$. Consequently $y=\frac{as}{1-s}$, and the Length of the Plane $\frac{cy^2}{s}$, where $c=16\frac{1}{12}$ Feet.

The same answered by Proteus.



Upon the Right-Line CD, perpendicular to the Horizon, let a Semicircle be described and make the Angle CDA equal to the given Angle: Join A, C and make AB perpendicular to CD, then will AB represent the Horizon, and AC the Plane. Let the given Interval of Time be denoted by *a*, the Natural Sine of the given Inclination by *s*, the Time of descending thro' CD by *T* and the Time of descending thro' CB by *t*; then by the Principles

ciples of Phyſicks $T^2 : t^2 :: CD : CB :: CD \times CB (AC^2) : CB^2$, whence $T : t :: AC : CB ::$ Rad. (1) : *Coſine* ACB (s); and $T : T-t (a) :: 1$: *Verſed Sine* ACB ($1-s$). Therefore $T = \frac{a}{1-s}$. Now if $16\frac{1}{12}$ the Diſtance perpendicularly deſcended by the Force of Gravity in the firſt Second of Time be denoted by d, we ſhall have $d : 1^2 :: CD : \frac{a^2}{1-s|^2}$; and $1 : s :: \frac{da^2}{1-s|^2} (CD) : \frac{sda^2}{1-s|^2} = AC$. Whence the Time of deſcending thro' CB will be found $= \frac{sa}{1-s}$.

This Problem was, alſo, anſwered by Meſſrs. R---ſon, Bevill, Widd, *and* Tommy Tangent.

PROBLEM LXXVIII. *Anſwered by* E. B—M.

By Articles 209 and 115 of Simpſon's Fluxions $\frac{2cs^3d^4}{12b^2r^4}$ is the Area of the Curve deſcribed by the Ball (in a Medium void of Reſiſtance) where s and c repreſent the Sine and Coſine of the Elevation; r, Unity; d the Space paſſed over in a given Time with an uniform Velocity, and b the Space deſcended by the Force of Gravity in the ſame Time. Therefore, by the Problem cs^3 (the other Quantities being conſtant) as alſo c^2s^6 will be a Maximum; but $1-s^2=c^2$, therefore s^6-s^8 a Maximum; which put into Fluxions and reduced gives $s=\sqrt{\frac{3}{4}}=$ *Sine* 60°. *Q. E. I.*

The same answered by Mr. R---son.

Let AB, perpendicular to the Horizontal Line AG, be the given Impetus, about which as a Diameter let a Circle be described cutting AH, the required Direction of the Ball to be projected, in C, and then CD, parallel to BA, will be its greatest Altitude, and AD equal to $\frac{1}{4}$ its Amplitude, by Art. 210, Simpson's Fluxions, Draw CE parallel to AG cutting the Circle in E and AB in L, and let the Points A, E, be joined; then, the Area described in the Flight, being constantly as

$AD \times DC (= LC \times AL = \frac{EC \times AL)}{2}$ will therefore be a Maximum when $\frac{EC \times AL}{2}$ (the Area of the Triangle EAC) is a Maximum, which will be when the Triangle is an Equilateral One, as appears from Theorem. 12th Pa. 114, Simpson's Geometry; and in that Circumstance the Angle of Elevation CAD will, it is evident, be $= 60°$.

This Problem was answered by Micromegas, *Mr.* Bevill, *and Mr.* Widd, *from the Assistance of Fluxions.*

PROBLEM

PROBLEM LXXIX. *Anſwered by Tommy Tangent.*

Let ESWN repreſent the Plane of the Horizon, E, W, N and S the Eaſt-Weſt North and South Points thereof, Z the Zenith and P the North Pole, R the Point where the Sun riſes and O his Place when the Shadow is projected towards R. Let the Arches PR and PO be drawn, then will the Angle OPR be equal to 165°, Let m, n expreſs the Sine and Coſine of Half the Angle OPR (82° 30′) and let the Sine of 51° 30′ the given Latitude be denoted by s, alſo that of PR (the Complement of the Sun's Declination at Riſing) by x; then the Declination being ſuppoſed to continue invariable for 11 Hours, or PR to be =PO, we ſhall have $1 : x :: m : mx$ the Sine of Half OR, whoſe Coſine is $\sqrt{1-m^2x^2}$; therefore $2mx\sqrt{1-m^2x^2}$ will be the Sine of OR, whence $2mx\sqrt{1-m^2x^2} : 2mn :: x : \frac{n}{\sqrt{1-m^2x^2}}$ the Sine of PRO; and $x : 1 :: s : \frac{s}{x}$ the Sine of PRN.

But theſe Angles (PRO and PRN) are the Complements of each other, therefore

$\frac{s^2}{x^2}+\frac{n^2}{1-m^2x^2}=1$, which reduced and tranſpoſed (by ſubſtituting m^2 for $1-n^2$) gives

$x^4-\overline{1+s^2}\times x^2=-\frac{s^2}{m^2}$; whence

$x=\sqrt{\frac{1+s^2}{2}+\sqrt{\frac{1+s^2}{2}\Big|^2-\frac{s^2}{m^2}}}=$, 985077 the Nat. Sine of 80° 5′ 19″. Conſequently the Sun's Declination was 9° 54′ 41″.

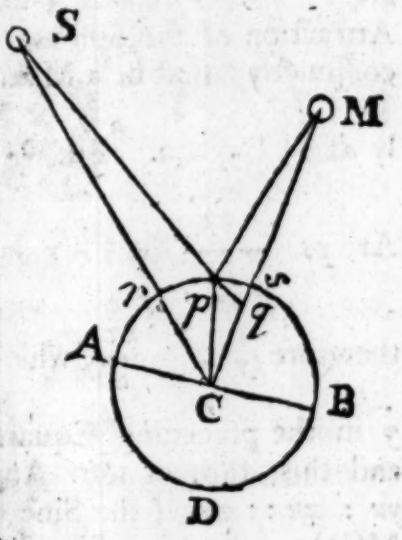
The ſame anſwered by E. B—M.

Put m and n for the Sine and Coſine of RPO the given Hour Angle, a and b for the Sine and Coſine of PZ (38° 30′), x and y for the Sine and Coſine of PR; then ſuppoſing PO=PR, we ſhall have $y \times \frac{1+n}{m}$ for the Cotangent of PRO. But in the preſent Caſe the Coſine of PRO will be expreſſed by $\frac{b}{x}$, therefore its Sine $\frac{\sqrt{x^2-b^2}}{x}$, and Cotangent $\frac{b}{\sqrt{x^2-b^2}} = \frac{b}{\sqrt{a^2-y^2}}$; Hence $y \times \frac{1+n}{m} = \frac{b}{\sqrt{a^2-y^2}}$, and from thence $2y^2 = a^2 - \sqrt{a^4 - \left|\frac{2bm}{1+n}\right|^2}$ the verſed Sine of (19° 50′ 8″) twice the Sun's Declination whoſe Half is 9° 55′ 4″.

Mr. R--ſon *anſwered this Problem, as did alſo Mr.* Widd, *who finds the Declination to be* 9° 57′ *North.*

PROBLEM

PROBLEM LXXX. *Anſwered by Mr.* R—ſon.



Let ABD repreſent the Earth, S the Sun, M the Moon, and the Angle SCM or *r*C*s* the given Poſition of the Sun and Moon at the Time propoſed.

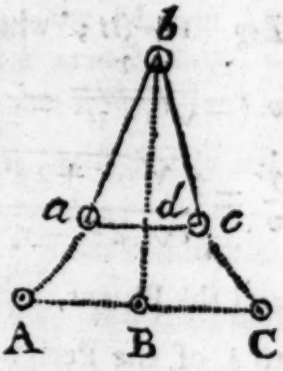
Suppoſe p repreſent the higheſt Point of the Ocean required; join the Points M, p and C, p and draw pq perpendicular to CM, and parallel thereto, thro' C the Center of the Earth, let AB be drawn.

Let r denote the Radius of the Earth, and d and D the Denſities of the Sun and Moon, which will alſo be as their Forces to produce Tides at the Earth's Surface (their apparent Diameters being nearly equal). Then, putting *x* and *y* for the Sine and Coſine of the Angle *p*C*q*, alſo *u* and *z* for the Sine and Coſine of the Angle *r*C*p* to Radius 1, the Force of the Moon upon the Particle p, in the Direction MC, will be as *rDy* (See Art. 403, Simpſon's Fluxions) which will alſo repreſent the Force in the Direction *p*M, ſince, on Account of the great Diſtance of the Moon from the Earth, CM and *p*M may be eſteemed parallel; whence, by the Reſolution of Forces, rDy^2 will be found equal to the Force whereby the Moon attracts the Particle p directly from the Earth's Center; and by reaſoning in the ſame Manner

Manner the Force whereby the Sun elevates the same Particle from the Center will be found to be drz^2 : Therefore $drz^2 + Dry^2$ (the whole Force of Attraction of the Sun and Moon on that Particle conjunctly) must be a Maximum, and consequently $dz\dot{z} + D\dot{y}y = 0$. But $-\frac{\dot{r}y}{x}$ (the Fluxion of the Arc ps) $-\frac{\dot{r}z}{u}$ (the Fluxion of the Arc pr) is $= 0$, therefore $\dot{y} = -\frac{\dot{x}z}{u}$; which being substituted for $\dot{y}$ in the preceding Equation gives $dzu = Dyx$; and this, thrown into Analogy, gives $d : D :: yx : zu :: 2yx$ (the Sine of Double the Angle MCp) : $2zu$ (the Sine of double the Angle SCp). Whence, by Composition and Division of Ratios, $D+d : D-d$ (:: Sine of Double the Angle SCp + Sine of Double the Angle MCp :: Sine of Double the Angle SCp — Sine of Double the Angle MCp) : : Tangent of the given Angle SCM : Tangent of the Difference of the Angles SCp, MCp by the Elements of Trigonometry, from whence the Angles themselves are readily determined.

PROBLEM

PROBLEM LXXXI. *Anſwered by Mr.* R--ſon.



Let A, B, C repreſent the Poſition of the Balls before the Impulſe, and *a*, *b*, *c* any other cotemporary Poſition of them after the Motion is begun, putting $ad\ (=dc) = x$, $bd = y$, and $ab\ (=bc) = a$. Put v for the Celerity whereby bd increaſes, and then $\frac{yv}{x} = tv$ (ſuppoſing $t = \frac{y}{x}$ = the Tangent of the Angle dab to Radius) will be Velocity by which ad decreaſes, and its Fluxion $\dot{t}v + v\dot{t}$ is the Increaſe of the ſaid Velocity, or the Effect of the Tenſion of the String in the Direction ad.

But the Motion, or the Velocity of the Ball b from the Line AC being denoted by v, $\frac{c-v}{3}$ will be the Velocity of that Line itſelf, whoſe Fluxion $-\frac{\dot{v}}{3}$ will conſequently be the Effects of the Tenſion of the String upon the Ball a in a Direction parallel to db. Moreover theſe Effects $\dot{tv} + v\dot{t}$ and $-\frac{\dot{v}}{2}$ of the Tenſion of the String in the Directions ad and db are, by the Reſolution of Forces, in the Ratio of ad to db, or of the Radius to the Tangent of the Angle dab; whence $\dot{tv} + v\dot{t} : -\frac{\dot{v}}{3} :: 1 : t$, and conſequently $-\frac{2\dot{v}}{v} = \frac{6tt}{1+3t^2}$,

and the Fluent corrected, 2 *Hyp. Log.* $\frac{c}{v} = Hyp.$ *Log.* $1+3tt$; whence will be found $v\ (=\frac{c}{\sqrt{1+3t^2}}) = c\sqrt{\frac{a^2-y^2}{a^2+2y^2}}$ and conſequently $\frac{\dot{y}}{v} = \frac{\dot{y}\sqrt{a^2+2y^2}}{c\sqrt{a^2-y^2}}$ = the Fluxion of the Time; and the Fluent, when $y=a$, viz. $\frac{1}{c}$ multiplied into $\frac{1}{4}$ of the Periphery of the Ellipſe, whoſe two Axes are $2a\sqrt{\frac{1}{3}}$ and $2a$, will expreſs the whole Time before the Congreſs of the two extreme Balls A and C in *d*.

But if the Balls were ſuppoſed to be perfectly elaſtic, and the two extreme ones A and C only equal, and each of them to the middle one in the Ratio of 1 to *n*, their Motion and Velocity in any Poſition after the firſt Colliſion might be from hence likewiſe determined; the Time they would be in Motion, before the two extreme Ones would firſt meet, being, in this Caſe, expreſſed by $\frac{1}{c}$ multiplied into $\frac{1}{4}$ of the Periphery of the Ellipſe whoſe Tranſverſe and Conjugate Diameters are $2a\sqrt{\frac{2+n}{n}}$ and $2a$ reſpectively.

The ſame anſwered by the Propoſer, Waltonienſis.

Let A, B, C be the Places of the Balls before the Impulſe; *a*, *b*, and *c* cotemporary Places of them after they are in motion. And, *d* being the Point of Interſection of the Lines B*b*, *ac*; call B*d*, *z*; *bd*, *x*; the Velocity of A or C in a Direction

rection parallel to Bb, v; their Velocity in a Direction parallel to AB or ad, u; the Tenſion of the String, S; and the Time the Motion has continued T. Then the Force accelerating the Velocity of A in a Direction parallel to Bb will be $\frac{Sx}{Aa}$; and that accelerating its Velocity in a Direction parallel to AB, $\frac{S\sqrt{a^2-x^2}}{Aa}$: Which Forces will be to each other as $\dot{v}$ to $\dot{u}$, and therefore $\frac{x}{\sqrt{a^2-x^2}} = \frac{\dot{v}}{\dot{u}}$. But $\dot{T}$ being expreſſed by either $\frac{\dot{z}}{v}$, $\frac{\dot{x}+\dot{z}}{c-2v}$, or $\frac{\dot{x}x}{u\sqrt{a^2-x^2}}$; thoſe Expreſſions will be equal amongſt Themſelves. By equating the two firſt $\dot{z} = \frac{v\dot{x}}{c-3v}$; and by equating the firſt and laſt $\dot{z} = \frac{v\dot{x}x}{u\sqrt{a^2-xx}}$; therefore $\frac{v\dot{x}}{c-3v} = \frac{v\dot{x}x}{u\sqrt{a^2-x^2}}$, conſequently $\frac{u}{c-3v} = \frac{x}{\sqrt{a^2-x^2}} = \frac{\dot{v}}{\dot{u}}$, $u\dot{u} = c\dot{v} - 3v\dot{v}$, $\frac{u^2}{2} = cv - \frac{3v^2}{2}$, and $u = \sqrt{2cv-3v^2}$. This Value of u being ſubſtituted for u itſelf, we have $\sqrt{\frac{2cv-3v^2}{c-3v}} = \frac{x}{\sqrt{a^2-x^2}}$; whence $v = \frac{1}{3}c - \frac{1}{3}c \times \sqrt{\frac{a^2-x^2}{a^2+2x^2}}$; conſequently $\frac{1}{3}\dot{x}\sqrt{\frac{a^2+2x^2}{a^2-x^2}} - \frac{1}{3}\dot{x} = \dot{z}$, and $\sqrt{\frac{a^2+2x^2}{a^2-x^2}} \times \frac{\dot{x}}{c} = \dot{T}$. The Relation of the Fluents of theſe flowing Quantities may be thus ſhewn.

Let ABC be a Quadrant of an Ellipsis; AC being a, BC$=3\frac{1}{2}a$; and let DE be parallel to CB. Then if x be denoted by CD, $\frac{BE}{c}$ will be equal to T, the Time the Balls have been in Motion; and $\frac{1}{3}$ BE$-\frac{1}{3}$ CD$=z$.

Coroll. The Accelerating Force into the Fluxion of the Time being equal to the Fluxion of the Velocity in the Direction in which that Force acts; we have $\frac{Sx}{Aa} \times \frac{\dot{z}}{v} = \dot{v}$, whence $S=\frac{Aav\dot{v}}{x\dot{z}}$. Substituting the Values of v, $\dot{v}$, and $\dot{z}$ found above, we have S (the String's Tension) equal to $\frac{Aa^3c^2}{a^2+2x^2|^2}$.

Schol. In this Solution Respect is only had to the Motion of the Balls before A and C meet. If those two Balls be perfectly hard, they will, after meeting, have no circulatory motion, but proceed directly after B, and all three will move with a Velocity equal to $\frac{1}{3}c$.

But if A and C be perfectly elastic they will, after their Congress, recede from each other with a circulatory Motion about B, till all three are in a Right-Line parallel to AC, when v will be equal to $\frac{2}{3}c$, and B will move backwards with a Celerity equal to $\frac{1}{3}c$. The circulatory Motion

of

of the two extreme Balls continuing they will again approach each other and meet; at which Time v will be $\frac{1}{3}c$, and B will move forward with a Velocity equal to the ſame Quantity; but will be behind A and C. After this ſecond Congreſs, thoſe two Balls will again recede from each other, 'till all three are again in a Right-Line; when the extreme Balls will have no Motion, and B will move forward, as at firſt, with the Celerity c, v will be equal $\frac{1}{3}c - \frac{1}{3}c \times \sqrt{\frac{a^2 - x^2}{a^2 + 2x^2}}$ and $\frac{1}{3}c + \frac{1}{3}c\sqrt{\frac{a^2 - x^2}{a^2 + 2x^2}}$ alternately; but the Relation of T, x and z will, during the whole Motion, be regularly determined by completing the Ellipſis and conceiving E to revolve in its Periphery, ſo that BE may denote the Space a Body would move in the Time T, with the uniform Velocity c.

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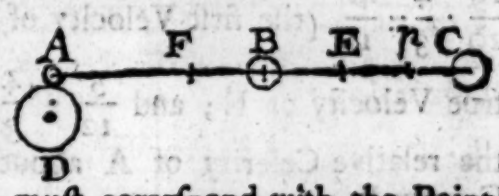
The Solution, given by Mr. *R---ſon*, to Problem 66 (ſee our laſt Number) is erroneous; however, as the Concluſion is not effected thereby, it might eaſily be corrected; but as he has ſent another Method of Solution which depends on common Principles, only, and is quite general, let the Number and Diſpoſition of the Balls be what they will, we have inſerted it and hope it will be acceptable to our Readers.

It is evident that the Point p, about which the Rod and Bodies



begin to move, muſt correſpond with the Point of

of Suspension, the Center of Percussion being in A, since the Point of Suspension of a Body is not affected by the Action of another Body at the Center of Percussion; and it is well known that, if the Center of Percussion of any Body be made the Point of Suspension, the former Point of Suspension will become the Center of Percussion of the Rod, supposing A to be the Point of Suspension.

This being premised, let E be the common Center of Gravity of all the Bodies, except D, and F the common Center of Gravity of them all including D; then the Velocity with which F begins to move will be to the given Velocity of D, as the Quantity of Matter in D to the Quantity of Matter in all the Bodies. Moreover it will be as $pF : pE$ :: the Velocity last named : the uniform Velocity of the Center of Gravity E; and lastly as $pE : EA$ :: the Velocity of the Center of Gravity E : the relative Velocity of the Point A about that Center.

In the Case proposed (where $A=1$, $B=2$, $C=3$, $D=4$, $AB=1$, and $AC=2$), we have, by the known Theorems, $AF=\frac{1\times2+2\times3}{1+4+2+3}=\frac{4}{5}$; $AE=\frac{1\times2+2\times3}{1+2+3}=\frac{4}{3}$; and $Ap=\frac{1\times2+4\times3}{1\times2+2\times3}=\frac{7}{4}$; whence $pF=\frac{19}{20}$ and $pE=\frac{5}{12}$. Then $\frac{19}{20} : \frac{4}{3} :: \frac{4c}{19}$ (the first Velocity of F) : $\frac{10c}{57}$, the true Velocity of E; and $\frac{5}{12} : \frac{4}{3} :: \frac{10c}{57} : \frac{32c}{57}$, the relative Celerity of A about the Center of Gravity E.

FINIS.

Part of No. II cf. titlepage

THE INVESTIGATION OF THE SUMS of SERIES's.

By *X A-K I A.*

LEMMA.

The Sum of the Geometrical Progreſſion $1 + x + x^2 + x^3 + x^4$, &c. *infinitely continued (ſuppoſing* x *leſs than Unity) is* $= \frac{1}{1-x}$.

FOR if the ſaid Progreſſion be multiplied by $1-x$, the Product will be

$$1-x \times 1 + x + x^2 + x^3 + x^4, \&c. =$$
$$\left\{\begin{array}{llllll} 1 & + x & + x & + x^3 & + x^4, & \&c. \\ * & - x & - x^2 & - x^3 & - x^4, & \&c. \end{array}\right\} = 1:$$

Whence, dividing, equally, by $1-x$, we have

$1 + x + x^2 + x^3 + x^4$, &c. $= \frac{1}{1-x}$. *Q. E. D.*

Problem I.

To find the Sum of the Infinite Series $x + 2x^2 + 3x^3 + 4x^4 + 5x^5$, &c.

By the Lemma, $\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4$, &c. The Fluxion of which gives $\frac{\dot{x}}{\overline{1-x}^2} = \dot{x} + 2x\dot{x} + 3x^2\dot{x} + 4x^3\dot{x}$, &c. and conſequently $\frac{x}{\overline{1-x}^2} = x + 2x^2 + 3x^3 + 4x^4 + 5x^5$, &c.

Q. E. I.

Problem II.

To find the Sum of the Infinite Series $x + 4x^2 + 9x^3 + 16x^4 + 25x^5$, &c.

It is evident, from Problem 1, that $\frac{x}{\overline{1-x}^2}$ $(= x \times \overline{1-x}^{-2}) = x + 2x^2 + 3x^3 + 4x^4$, &c. which, in Fluxions, gives $\dot{x} \times \overline{1-x}^{-2} + 2x\dot{x} \times \overline{1-x}^{-3} = \dot{x} + 4x\dot{x} + 9x^2\dot{x} + 16x^3\dot{x}$, &c. From whence, by Reduction $\frac{x+x^2}{\overline{1-x}^3} = x + 4x^2 + 9x^3 + 16x^4$, &c. *Q. E. I.*

Problem III.

To find the Sum of the infinite Series $x + 8x^2 + 27x^3 + 64x^4$, &c.

By the laſt Problem $\overline{x+x^2} \times \overline{1-x}^{-3} = x + 4x^2 + 9x^3 + 16x^4$, &c. The Fluxion whereof (making $\dot{x}$ = Unity) is $\overline{1+2x} \times \overline{1-x}^{-3} + \overline{3x+3x^2} \times \overline{1-x}^{-4} = 1 + 8x + 27x^2 +$

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$64\,x^3 + 125\,x^4$, &c. Therefore by Reduction, $\frac{x + 4\,x^2 + x^3}{\overline{1-x}|^4} = x + 8\,x^2 + 27\,x^3 + 64\,x^4$, &c.

Q. E. I.

By the ſame Method it will appear that $\frac{x + 11\,x^2 + 11\,x^3 + x^4}{\overline{1-x}|^5} = x + 2^4\,x^2 + 3^4\,x^3 + 4^4\,x^4 + 5^4\,x^5$, &c.

$\frac{x + 26\,x^2 + 66\,x^3 + 26\,x^4 + x^5}{\overline{1-x}|^6} = 1 + 2^5\,x^2 + 3^5\,x^3 + 4^5\,x^4 + 5^5\,x^5$, &c.

&c.

Where, as well as in the preceding Caſes, the Value of x is ſuppoſed leſs than Unity: (The Sum of the Series being infinite, when it is equal to, or greater than Unity.)

It may alſo be proper to take Notice here, that this Method Anſwers with equal Facility, when the Factors to each Term of the Series are unequal, as in the Series $1 \times 10\,x^4 + 2 \times 11\,x^5 + 3 \times 12\,x^6 + 4 \times 13\,x^7$, &c.

For, if the Equation $\frac{x}{\overline{1-x}|^2} = x + 2\,x^2 + 3\,x^3 + 4\,x^4$, &c. (given by Prob. 1.) be multiplied by x^9 (ſo that the Exponent of the firſt Term of this Series may be equal to the ſecond (or new) Factor in the firſt Term of the given Series) then will $\frac{x^{10}}{\overline{1-x}|^2}$ (or $x^{10} \times \overline{1-x}|^{-2}$) $= x^{10} + 2\,x^{11} + 3\,x^{12} + 4\,x^{13}$, &c. whoſe Fluxion gives $10\,x^9 \times \overline{1-x}|^{-2} + 2\,x^{10} \times \overline{1-x}|^{-3} = 10\,x^9 + 2 \times 11\,x^{10} + 3 \times 12\,x^{11}$, &c. whence $\frac{10\,x^4 - 8\,x^5}{\overline{1-x}|^3} = 1 \times 10\,x^4 + 2 \times 11\,x^5 + 3 \times 12\,x^6 +$, &c. Moreover from hence the Value of the Series

$10z^4 - 2 \times 11z^5 + 3 \times 12z^6 - 4 \times 13z^7$, &c. (where the Signs are + and — alternately) may be eaſily deduced: For, making $z = -x$, the ſaid Series will become $(= 10x^4 + 2 \times 11x^5 + 3 \times 12x^6$, &c. $= \frac{10x^4 - 8x^5}{\overline{1-x}|^3}) = \frac{10z^4 + 8z^5}{\overline{1+z}|^3}$.

And by the ſame Method the Values of $x - 2x^2 + 3x^3 - 4x^4$, &c. and $x - 4x^2 + 9x^3 + 16x^4$, &c. may alſo be found.

Problem IV.

To find the Sum of any Number (n) *of Terms of the Series* $x + x^2 + x^3 + x^4$, &c.

It is evident from the Law of the Series, that, when the n firſt Terms are taken away, the remaining Terms will be $x^{n+1} + x^{n+2} + x^{n+3} + x^{n+4}$, &c. $= x^{n+1} \times \overline{1 + x + x^2 + x^3 + x^4}$, &c. *ad Infinitum* $= x^{n+1} \times \frac{1}{1-x}$ (*p.* Lemma): Which deducted from $\left(\frac{1}{1-x}\right)$ the Value of the whole Series $(x + x^2 + x^3$, &c.) leaves $\frac{x - x^{n+1}}{1-x} = x + x^2 + x^3 \text{ ---- } + x^n$. *Q. E. I.*

Problem V.

To find the Sum of n *Terms of the Series* $x + 2x^2 + 3x^3 + 4x^4$, &c.

The laſt Term to be here taken being nx^n, the remaining Part of the Series will therefore be $\overline{n+1} \times x^{n+1} + \overline{n+2} \times x^{n+2} + \overline{n+3} \times x^{n+3}$, &c. which may be reſolved into

$nx^n \times \overline{x + x^2 + x^3 + x^4 + x^5}$, &c. $= \frac{nx^{n+1}}{1-x}$ (*p.* Lem.)

and $x^n \times \overline{x + 2x^2 + 3x^3 + 4x^4}$, &c. $= \frac{x^{n+1}}{\overline{1-x}|^2}$ (*p.* Prob. 1)

The

The Sum of which, ſubſtracted from $\frac{x}{\overline{1-x}|^2}$ $(=x+2x^2+3x^3$,&c.) gives $\frac{x-\overline{n+1}-nx\times x^{n+1}}{\overline{1-x}|^2}$ $=x+2x^2+3x^3$ ----- $+nx^n$. *Q. E. I.*

Problem VI.

To find the Sum of n *Terms of the Series* $x+4x^2+9x^3+16x^4$, &c.

Here the remaining Terms, after the n firſt are taken away, will be $\overline{n+1}|^2\times x^{n+1}+\overline{n+2}|^2\times x^{n+2}+\overline{n+3}|^2\times x^{n+3}$, &c.

$$=\left\{\begin{array}{r} n^2x^n\times\overline{x+x^2+x^3+x^4,\ \&c.} \\ 2nx^n\times\overline{x+2x^2+3x^3+4x^4,\&c.} \\ x^n\times\overline{x+4x^2+9x^3+16x^4,\&c.} \end{array}\right\}=$$

$\frac{n^2x^{n+1}}{1-x}+\frac{2nx^{n+1}}{\overline{1-x}|^2}+\frac{1+x\times x^{n+1}}{\overline{1-x}|^3}$ (*by Prob.* 1 *and* 2)

which deducted from $\frac{x+x^2}{\overline{1-x}|^3}$ leaves $\frac{x+x^2}{\overline{1-x}|^3}-\frac{x^{n+1}}{1-x}\times$

$n^2+\frac{2n}{1-x}+\frac{1+x}{\overline{1-x}|^2}=x+4x^2+9x^3$ -----$+n^2x^n$.

Q. E. I.

To be continued in next Number.

THE PRINCIPLES OF DIALLING.

DEFINITIONS.

1. A Dial is a Plane upon which Lines are deſcribed in ſuch Sort, that the Shadow of the upper Edge of another Plane, erected perpendicular thereto, may point out the Hours of the Day.

2. The Edge of the ſaid Plane, by which the Hours of the Day are marked out, is called the Style of the Dial and is, *always*, parallel to the Axis of the Earth.

3. The Line in which the ſaid Plane interſects the Plane of the Dial is called the Subſtyle.

4. The Angle included between the Style and Subſtyle is called the Height of the Style.

The Poſition of any Dial is diſtinguiſhed according as it is either Parallel, Perpendicular or Oblique to the Plane of the Horizon of that Place where it is fixed.

5. Thoſe Dials, whoſe Planes are parallel to the Plane of the Horizon, are called Horizontal Dials.

6. Thoſe Dials, whoſe Planes are perpendicular to the Plane of the Horizon, are called Erect Dials.

7. Thoſe Erect Dials, whoſe Planes are either parallel or perpendicular to the Plane of the Meridian, are called Direct Erect Dials; becauſe they front the four Cardinal Points.

8. All other Erect Dials are called Declining Dials.

9. Thoſe Dials, whoſe Planes are neither parallel nor perpendicular to the Plane of the Horizon, are called Inclining or Reclining Dials.

10.

10. Thoſe Inclining or Reclining Dials, whoſe Planes are either perpendicular to the Meridian or Prime Vertical, are called Direct Inclining or Reclining Dials.

11. All thoſe Oblique Dials, whoſe Planes make an obtuſe Angle with the Horizon, are called Reclining Dials; and thoſe, whoſe Planes make an acute Angle with the Horizon, are called Inclining Dials.

12. The Arch of the Horizon, which is intercepted between any given Plane and That of the Prime Vertical, is ſaid to be the Declination of that Plane.

13. The Inclination of the Plane is the Angle which the Plane makes with the Horizon.

14. The Reclination of the Plane is the Exceſs of the Angle, which the Plane makes with the Horizon, above a Right Angle.

15. The Interſection of the Plane of the Dial and that of the Meridian, paſſing thro' the Style, is called the Meridian of the Dial, or the Hour Line of 12.

16. Thoſe Meridians, paſſing thro' the Style, which make Angles of 15°, 30°, 45°, *&c.* with the Meridian of the Place (marking the Hour Line of 12) are called Hour Circles; and their Interſections, with the Plane of the Dial, Hour Lines.

17. The Angle, formed by the Subſtyle and the Hour Line of 12, is called the Diſtance of the Subſtyle from the Meridian.

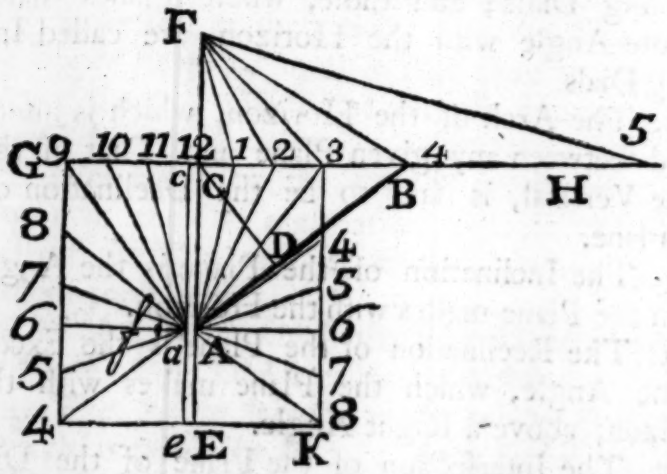
18. The Plane's Difference of Longitude is the Angle formed at the Style by two Meridians; one paſſing thro' the Hour Line of 12, and the other thro' the Subſtyle.

PROB-

Problem I.

To deſcribe an Horizontal Dial, Geometrically, for any propoſed Latitude.

Construction.



On the propoſed Plane draw the Right Line 6A6 for the Eaſt and Weſt Line of the Dial, or the Hour Line of Six; in the Middle of which ſet off Aa equal to the propoſed Thickneſs of your Style; thro' A, a draw CAE, cae both perpendicular to 6A6, and let GCH be parallel thereto; make the Angle CAB equal to the given Latitude, and upon AB let fall the Perpendicular CD; in AC produced take CF equal to CD, then make the Angles CF1, CF2, CF3, &c. equal to 15°, 30°, 45°, &c. reſpectively, and draw A1, A2, A3, &c. for the Hour-Lines of One, Two, Three, &c. after Noon. Moreover, take c11, c10, c9, &c. reſpectively, equal to C1, C2, C3, &c. and draw a11, a10, a9, &c. for the Hour-Lines of Eleven, Ten, Nine, &c. before Noon.

The Reader is deſired to obſerve that the Line drawn from F to B, in the Cut, ought to have been drawn ſo as to meet the Line A 4, produced, in the Line GH.

Note, The Hour-Lines of Four and Five, whoſe Interſections with GCH fall without the Plane of the Dial, may be otherwiſe drawn in the following Manner. Take A*f* equal to the Exceſs of AC above CD, and make the Angles 6*f*5, 6*f*4 equal to 15°, 30° reſpectively, alſo from the Points 4, 5 where the Lines *f*4, *f*5 interſect K3, parallel to CE, draw A4, A5 for the Hour-Lines of Four and Five; from which, by taking 6, 7 and 6, 8 equal to 6, 5 and 6, 4, *&c.* and drawing A7, A8, *&c.* you will alſo have the Hour-Lines of Seven and Eight.

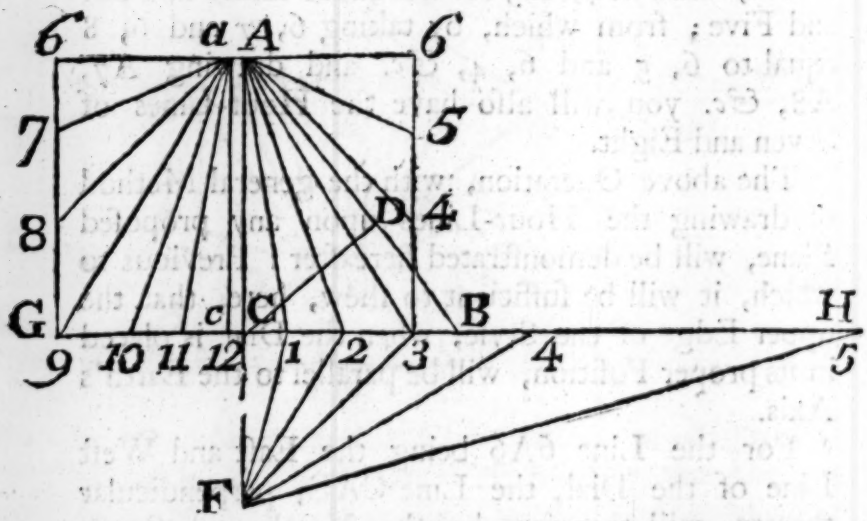
The above Operation, with the general Method of drawing the Hour-Lines upon any propoſed Plane, will be demonſtrated hereafter: Previous to which, it will be ſufficient to ſhew, here, that the upper Edge of the Style, when the Dial is placed in its proper Poſition, will be parallel to the Earth's Axis.

For the Line 6A6 being the Eaſt and Weſt Line of the Dial, the Line CAE, perpendicular thereto, will therefore be the North and South Line with Regard to the Center A, or in Reſpect of that Side of the Style, by the Shadow of which the Hours after Noon are pointed out; and it is evident that, as A*a* expreſſes the Thickneſs of the Style, the Line *cae* will be the North and South Line of the Dial, with Regard to the Center *a*, anſwering to the Morning Side of the Style. Moreover, ſince the Angle CAB is equal to the Latitude of the Place or the Elevation of the Pole, by Conſtruction, it is evident, that when the Style CAB is erected perpendicular to the Plane of the Horizon, on A*ac*C, the upper Edge thereof will paſs thro' the Pole of the World and ſo be parallel to the Earth's Axis.

Problem II.

To deſcribe an Erect South Dial, Geometrically, for any given North Latitude; or an Erect North Dial for any South Latitude.

Construction.



On the propoſed Plane, draw the Right-Line 6A6 for the Eaſt and Weſt Line of the Dial, or the Hour Line of Six; in the Middle of which ſet off A*a* equal to the propoſed Thickneſs of your Style, and from A, *a* draw AC, *ac* both perpendicular to 6A6, alſo let GCH be parallel thereto: Make the Angle CAB equal to the Complement of the Latitude, and upon AB let fall the Perpendicular CD, then proceed as in Prob. I.

To ſhew that the upper Edge of the Style, when the Dial is placed in its proper Poſition, will be parallel to the Earth's Axis (as it ought to be by Definition II.) we may obſerve, in the firſt Place, that, the Line 6A6 being the Eaſt and Weſt Line of the Dial, and parallel to the Plane of the Horizon, the Line AC perpendicular thereto (when the

the Plane of the Dial is erect according to Definition VI.) will alſo be perpendicular to the Horizon. Therefore when the Plane of the Style, repreſented by the Triangle ACB, is erected perpendicular to the Plane of the Dial, on the Line AC, it will coincide with the Meridian (being perpendicular to the Plane of the Dial repreſented by the Prime Vertical:) And ſo the Angle CBA being equal to the given Latitude, or the Elevation of the Pole (by Conſtruction) it is evident AB the upper Edge of the Style, when produced, will paſs thro' the Pole of the World, and ſo be parallel to the Earth's Axis.

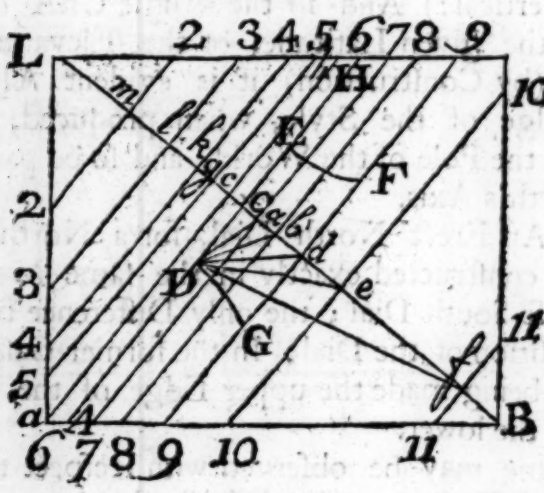
Note, An Erect North Dial, for a North Latitude, is conſtructed exactly in the ſame Manner as an Erect South Dial; the only Difference being in the Poſition of the Dial: In the former Caſe the Line GB being made the upper Edge of the Dial inſtead of the lower.

The ſame may be obſerved with Reſpect to an Erect South Dial, for a South Latitude.

Problem III.

To describe an Erect East Dial, Geometrically, for any proposed North Latitude; or an Erect West Dial for any South Latitude.

Construction.



On the proposed Plane draw the Right-Line AB for the Horizontal Line of the Dial; make the Angle LBA equal to the Complement of Latitude, and, perpendicular to BL, draw ACH for the Hour-Line of Six; take C*c* equal to the proposed Thickness of your Style, and parallel to AH draw *ach*; then, having assumed DC at pleasure, make the Angles CD*a*, CD*b*, CD*d*, &*c*. equal to 15°, 30°, 45°, &*c*. respectively, and thro' the Points *a*, *b*, *d*, &c. parallel to ACH, draw the Lines 7*a*7, 8*b*8, 9*d*9, &c. for the Hour-Lines of Seven, Eight, Nine, &*c*. Moreover, take *cg*, *ck*, *cl*, &c. equal to C*a*, C*b*, C*d*, &c. and thro' *g*, *k*, *l*, &c. parallel to *ach*, draw the Lines 5*g*5, 4*k*4, 3*l*3, &c. for the Hour-Lines of Five, Four, Three, &*c*. The Style in this Case will be represented by the Plane H99A or DGFE, whose upper

per Edge GF produced will paſs thro' the Pole of the World (as it ought by Definition II.)

For the Dial being placed in its proper Poſition, the Plane thereof will coincide with the Meridian (by Definition VII.) and the Line BA with the Horizon; whence, the Angle BAC being equal to the propoſed Elevation of the Pole by Conſtruction, the Line ACH will, it is evident, point directly to the ſaid Pole, and therefore be parallel to the Earth's Axis. Therefore, GF being parallel to AH, it muſt alſo be parallel to the Earth's Axis.

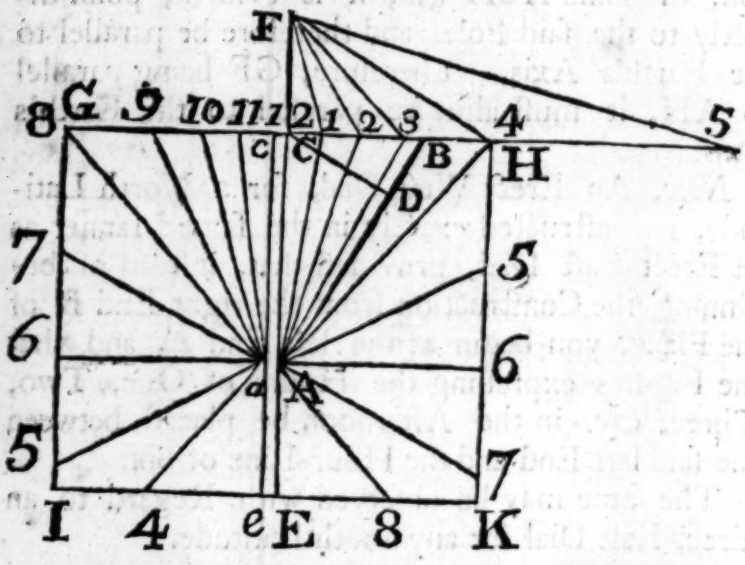
Note, An Erect Weſt Dial, for a North Latitude, is conſtructed exactly in the ſame Manner as an Erect Eaſt Dial, provided that, inſtead of beginning the Conſtruction from the right End B of the Plane, you begin at the left End A, and that the Figures expreſſing the Hours of One, Two, Three, *&c.* in the Afternoon be placed between the ſaid left End and the Hour-Line of Six.

The ſame may be obſerved with Regard to an Erect Eaſt Dial for any South Latitude.

Problem IV.

To describe a direct South Reclining Dial, Geometrically, for any North Latitude; or a direct North Reclining Dial, for any South Latitude.

Construction.



On the propoſed Plane draw the Right Line 6A6 for the Eaſt and Weſt Line of the Dial, or the Hour-Line of Six; in the Middle of which ſet off A*a* equal to the propoſed Thickneſs of your Style; thro' A, *a* draw CAE, *cae* both perpendicular to 6A6, and let GCH be parallel thereto; make the Angle CAB equal to the Exceſs of the Reclination above the Complement of Latitude, and upon AB let fall the Perpendicular CD; in AC produced take CF equal to CD, then proceed as in the firſt Problem.

To demonſtrate that the upper Edge of the Style, when the Dial is placed in its proper Poſition, will be parallel to the Earth's Axis (as it ought to be by Definition II.) we may obſerve in the firſt Place, that, the Line 6A6, being the Eaſt and

and Weſt Line of the Dial and parallel to the Horizon, will be perpendicular to the Plane of the Meridian; whence, the Line CAE, perpendicular to 6A6, will be the Meridian Line of the Dial, by Definition XV; and the Plane of the Style, ACB, when erected perpendicular to the Plane of the Dial, on the Line AC, will coincide with the Plane of the Meridian. Moreover, the Angle CAB being here equal to the Exceſs of the Reclination above the Pole's Diſtance from the Zenith, it is evident that the Difference between the Reclination and the ſaid Angle CAB will be equal to the Diſtance of the Pole from the Zenith, and conſequently AB, when produced, will paſs thro' the Pole, and ſo be parallel to the Earth's Axis.

It may be obſerved that, in the foregoing Conſtruction, the Pole elevated above the Horizon of the Place is alſo elevated above the Plane of the Dial: But, when the Reclination is leſs than the Poles Diſtance from the Zenith, the oppoſite Pole will be elevated; in which Caſe, the Height of the Style (CAB) muſt be ſet off, on the contrary Side of the Line 6A6, from the lower Part of the Meridian AE, which will then be the Hour-Line of Twelve.

If it be required to deſcribe a North Reclining Dial for a North Latitude, or a South Reclining One for a South Latitude, the Method of Conſtruction will ſtill be the ſame, *only*, inſtead of taking the Angle CAB equal to the Difference between the Reclination and the Complement of Latitude, you muſt take it equal to the Sum of them.

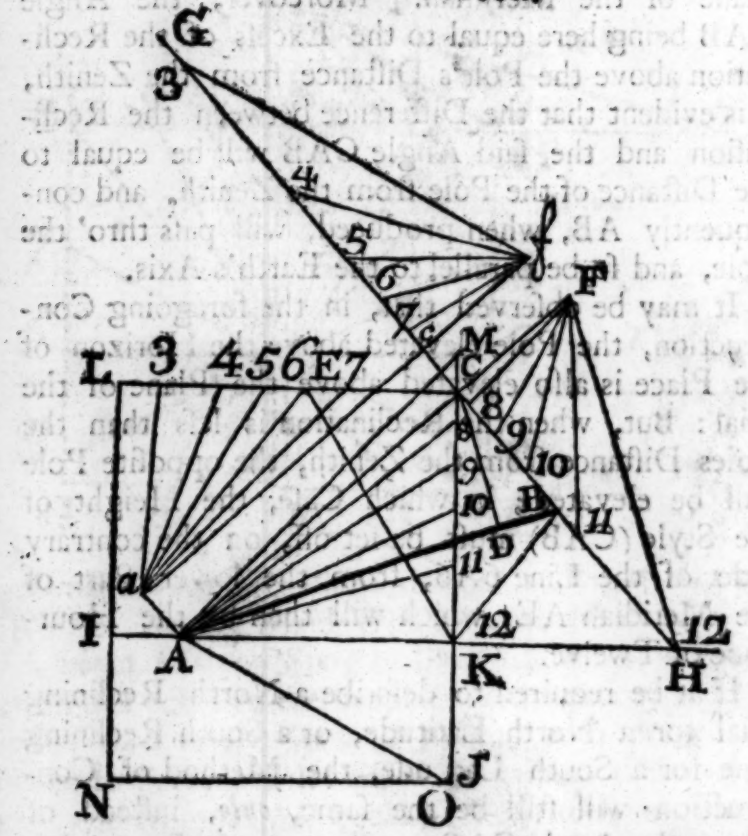
If the Plane of a North Reclining Dial be inverted, the Dial will then become a South Inclining Dial, whoſe Inclination is the Complement of the Reclination of the Dial ſo inverted.

In like Manner, a South Reclining Dial will become a North Inclining one.

Problem V.

To describe a direct East Reclining Dial, Geometrically, for any North Latitude; or a direct West Reclining Dial for any South Latitude.

Construction.



On the proposed Plane draw the Right-Line IK12 for the North and South Line of the Dial, or the Hour-Line of 12, perpendicular to which draw KC: Make EKC equal to the given Reclination, and perpendicular to KC draw CEL; take KM=KE and make the Angle AMK equal to the Complement of the proposed Latitude; join A, C and perpendicular thereto draw GCH, in which set off C*c* equal to the proposed Thickness of your Style;

Style; thro' *c* draw *ca* perpendicular to GH and let A*a* be parallel thereto: Make CB=CE, and joining A, B and K, B, draw CD perpendicular to AB; in AC and *ac*, produced, take CF and *cf* each equal to CD, and from F, to the Point where GH interſects IK, draw F12; then make the Angles 12F11, 11F10, 10F9, 9F8 each equal to 15°, and the Angle *tf*7 equal to the Exceſs of 15° above the Angle CF8: Alſo make the Angles 7*f*6, 6*f*5, *&c.* each equal to 15°, and draw the Right Lines A11, A10, A9, *&c.* for the Hour Lines of 11, 10, 9, *&c.*

To ſhew that the upper Edge of the Style, when the Dial is placed in its proper Poſition, will be parallel to the Earth's Axis (as it ought to be by Definition II.) we may obſerve in the firſt Place, that the Plane of the Triangle KEC, when erected perpendicular to the Plane of the Dial on the Line KC, will be parallel to the Prime Vertical (by Definition X.) and perpendicular to the Line IK, which, being the North and South Line of the Dial, is parallel to the Planes of the Horizon and of the Meridian, and therefore, becauſe the Angle EKC expreſſes the Reclination of the Plane of the Dial from the Plane of the Meridian, the Line EK will be perpendicular to the Plane of the Horizon, and parallel to the Plane of the Meridian.

Now the Triangles KEC, KBC, having the Side EC=BC, KC common, and the Angles ECK, BCK both right, will be equal and ſimilar; therefore when the Plane of the Style, repreſented by the Triangle ACB, and the Plane of the Triangle KBC are erected perpendicular to the Plane of the Dial, on the Lines AC, KC reſpectively, the Triangle AKB, as the Triangles KEC, KBC then coincide, will be parallel to the Plane of the Meridian. Moreover, becauſe BK (=KE) =KM, AK common, and the Angles AKM, AKB both

right, the Triangles AKM, AKB will be equal and ſimilar, and therefore ABK = AMK = the Complement of the given Latitude by Conſtruction; conſequently, the Angle KAB being equal to the given Latitude, the upper Edge of the Style AB, when produced, will paſs thro' the Pole of the World, and ſo be parallel to the Earth's Axis.

If it be required to deſcribe a Weſt Reclining Dial for a North Latitude, or an Eaſt Reclining One for a South Latitude, the Method of Conſtruction will ſtill be the ſame; provided that, inſtead of beginning the Operation towards the Right End of the Line IK, you begin towards the Left End, and that the Figures expreſſing the Hours after Noon be placed, inſtead of thoſe expreſſing the Hours before Noon.

A direct Weſt ~~Reclining~~ Dial, for a North Latitude, is conſtructed in the ſame Manner as a direct Eaſt Reclining One, only that the Points A and C change their Poſition (A being placed, in this Caſe, at the left End of the Line LC, and C at the Right End of the Line IK) and that (the Sun not coming on the Plane till after Noon) the Figures expreſſing the Hours after Noon be placed inſtead of thoſe before Noon.

The ſame may be obſerved with Regard to a direct Eaſt ~~Reclining~~ Dial for a South Latitude.

Continued in Page 24.

The INVESTIGATION of the SUMS of SERIES's continued.

After the ſame Manner the Sum of the n firſt Terms of the Series $x + 8x^2 + 27x^3 + 64x^4$, &c. will be found $= \frac{x+4x^2+x^3}{\overline{1-x}|^4} - \frac{x^{n+1}}{1-x} \times n^3 + \frac{3n^2}{1-x} + \frac{3n \times 1+x}{\overline{1-x}|^2} + \frac{1+4x+x^2}{\overline{1-x}|^3}$: And ſo of Others.

PROBLEM VII.

To find the Sum of the Infinite Series $ay^m + \overline{a+d} \times y^{m+n} + \overline{a+2d} \times y^{m+2n} + \overline{a+3d} \times y^{m+3n} +$ &c.

The Series propoſed, may be reſolved into theſe two others, *viz.*

$ay^m + ay^{m+n} + ay^{m+2n} + ay^{m+3n} +$, &c.

$dy^{m+n} + 2dy^{m+2n} + 3dy^{m+3n} +$, &c.

Whereof the Former (or its Equal $ay^m \times 1 + y^n + y^{2n} + y^{3n} +$, &c.) by making $x = y^n$, will become $ay^m \times 1 + x + x^2 + x^3 + x^4 +$, &c. $= ay^m \times \frac{1}{1-x}$ (by the Lemma) $= ay^m \times \frac{1}{1-y^n} = \frac{ay^m}{1-y^n}$.

Moreover the Latter is, in like Manner, reduced to $dy^m \times x + 2x^2 + 3x^3 + 4x^4 +$, &c. Whoſe Sum, *by Prob.* 1. is $dy^m \times \frac{x}{\overline{1-x}|^2} = \frac{dy^{m+n}}{\overline{1-y^n}|^2}$.

Therefore $\frac{ay^m}{1-y^n} + \frac{dy^{m+n}}{\overline{1-y^n}|^2}$ is the true Value which was to be determined.

Problem VIII.

To find the Sum of the Infinite Series $\overline{aby^m} + \overline{a+p} \times \overline{b+q} \times y^{m+n} + \overline{a+2p} \times \overline{b+2q} \times y^{m+2n} + \overline{a+3p} \times \overline{b+3q} \times y^{m+3n} +$, &c.

The Series is (by Multiplication) reſolved into the three following Ones, *viz.*

$$\overline{aby^m} \times 1 + y^n + y^{2n} + y^{3n} + y^{4n} +, \&c.$$
$$\overline{aq + bp} \times y^m \times y^n + 2y^{2n} + 3y^{3n} + 4y^{4n} +, \&c.$$
$$pqy^m \times y^n + 4y^{2n} + 9y^{3n} + 16y^{4n} +, \&c.$$

Whereof the Sums are $\frac{aby^m}{1-y^n}$, $\frac{\overline{ap+bq} \times y^{m+n}}{\overline{1-y^n}^2}$, and $\frac{pq \times \overline{y^n + y^{2n}}}{\overline{1-y^n}^3}$, reſpectively (*by Lem. and Prob.* 1 and 2.) Therefore the Aggregate of all Theſe is the true Value ſought.

After the ſame Manner the Sum of the Infinite Series $abcy^m + \overline{a+p} \times \overline{b+q} \times \overline{c+r} \times y^{m+n} + \overline{a+2p} \times \overline{b+2q} \times \overline{c+2r} \times y^{m+2n} +$, &c. &c. will be found, &c. &c.

Having exhibited a Method for finding the Sums of ſuch Series's as can be truly expreſſed in Algebraic Terms (by Means of the direct Method of Fluxions) I ſhall now proceed to ſhew how the Values of other Kinds of Series's may be determined, by Help of the Inverſe Method.

Pro-

$\left(\dot{y} = \frac{\dot{x}}{1-x} = \frac{-\dot{x}}{x-1}\right)$

Problem IX.

The Sum of the Infinite Series $x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} +$ &c. (= Fluent $\frac{\dot{x}}{1-x}$ = *Hyp. Log.* $\frac{1}{1-x}$ *being ſuppoſed given* $(= y:)$ *to find the Sum of the Infinite Series* $\frac{x}{1.2} + \frac{x^2}{2.3} + \frac{x^3}{3.4} + \frac{x^4}{4.5} +$, &c.

Since $y = x + \frac{x^2}{2} + \frac{x^3}{3} +$, &c. We have $y\dot{x} = x\dot{x} + \frac{x^2\dot{x}}{2} + \frac{x^3\dot{x}}{3}$ &c. whoſe Fluent is yx — Fluent of $x\dot{y} = \frac{x^2}{2} + \frac{x^3}{2.3} + \frac{x^4}{3.4}$, &c. But $x\dot{y} = x \times \frac{\dot{x}}{1-x} = \frac{x\dot{x}}{1-x} = -\dot{x} + \frac{\dot{x}}{1-x}$ (by Diviſion) whoſe Fluent is therefore $= -x + y$: Conſequently $yx + x - y = \frac{x^2}{1.2} + \frac{x^3}{2.3} + \frac{x^4}{3.4}$, &c. or $\frac{x - y \times 1 - x}{x} = \frac{x}{1.2} + \frac{x^2}{2.3} + \frac{x^3}{3.4}$, &c. $\left(= \frac{x+y \cdot x-1}{x}\right)$

Q. E. F.

Corollary. If $x = 1$, the Equation will become $1 = \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} +$, &c. and then the hyp. log of $\frac{1}{1-x}\left(\frac{1}{0}\right) = y$ will be infinite; in which Caſe $yx + x - y = x = \frac{1}{2} + \frac{1}{2.3} + \frac{1}{3.4} +$ &c. $= 1$.

Pro-

Problem X.

The ſame being ſuppoſed as in the laſt Problem: To find the Sum (S) *of the Series* $\frac{x}{1.2.3} + \frac{x^2}{2.3.4} + \frac{x^3}{3.4.5} + \frac{x^4}{4.5.6}$, *&c.*

By the laſt Problem $x\dot{x} + y \times \overline{x-1} \times \dot{x} = \frac{x^2\dot{x}}{1.2} + \frac{x^3\dot{x}}{2.3} + \frac{x^4\dot{x}}{3.4}$, *&c.* From which, by taking the Fluent, we get $\frac{x^2}{2} + \frac{y \times \overline{x-1}|^2-1}{2}$ — Fluent $\dot{y} \times \frac{\overline{x-1}|^2-1}{2} = \frac{x^3}{1.2.3} + \frac{x^4}{2.3.4} + \frac{x^5}{3.4.5}$, *&c.* = Sx^2. But $\dot{y} \times \frac{\overline{x-1}|^2-1}{2} = \frac{\dot{x}}{1-x} \times \frac{\overline{x-1}|^2-1}{2} = \frac{\dot{x} \times \overline{1-x}}{2} - \frac{\frac{1}{2}\dot{x}}{1-x}$; and its Fluent $= \frac{x}{2} - \frac{x^2}{4} - \frac{y}{2}$. Therefore $\frac{3x^2-2x+2y \times \overline{1-x}|^2}{4x^2} = S$.

Q. E. I.

Corollary. If $x = 1$, the propoſed Series will become $\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} +$, *&c.* = $\frac{1}{4}$.

Pro-

Problem XI.

The ſame being ſti'l ſuppoſed: To find the Sum (S) *of the Series* $\frac{x}{1.n+1} + \frac{x^2}{2.n+2} + \frac{x^3}{3.n+3} + \frac{x^4}{4.n+4}$ *&c. (where* n *denotes any whole poſitive Number.*

Since $y = \frac{x}{1} + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4}$, if both Sides of this Equation be multiplied by $x^{n-1}\dot{x}$, we ſhall have $yx^{n-1}\dot{x} = \frac{x^n\dot{x}}{1} + \frac{x^{n+1}\dot{x}}{2} + \frac{x^{n+2}\dot{x}}{3}$, &c. Whereof the Fluent is $\frac{yx^n}{n}$ — Fluent $\frac{\dot{y}x^n}{n} = \frac{x^{n+1}}{1.n+1} + \frac{x^{n+2}}{2.n+2} + \frac{x^{n+3}}{3.n+3}$, &c. $= Sx^n$. But $-\dot{y}x^n = -\frac{x^n\dot{x}}{1-x} = \frac{x^n\dot{x}}{x-1} = x^{n-1}\dot{x} + x^{n-2}\dot{x} + x^{n-3}\dot{x} + \text{------} \dot{x} + \frac{\dot{x}}{x-1}$ (by Diviſion) and therefore its Fluent $= \frac{x^n}{n} + \frac{x^{n-1}}{n-1} + \frac{x^{n-2}}{n-2}$ ----- $+ \frac{x}{1} - y$. Conſequently $\frac{yx^n}{n} + \frac{x^n}{n^2} + \frac{x^{n-1}}{n.n-1} + \frac{x^{n-2}}{n.n-2}$ ----- $+ \frac{x}{n} - \frac{y}{n} = Sx^n$, and $S = \frac{1}{n}$ into $y \times 1 - x^{-n} + \frac{1}{n} + \frac{1}{n-1.x} + \frac{1}{n-1.x^2} + \frac{1}{n-3.x}$ --- $\frac{1}{x^n}$.

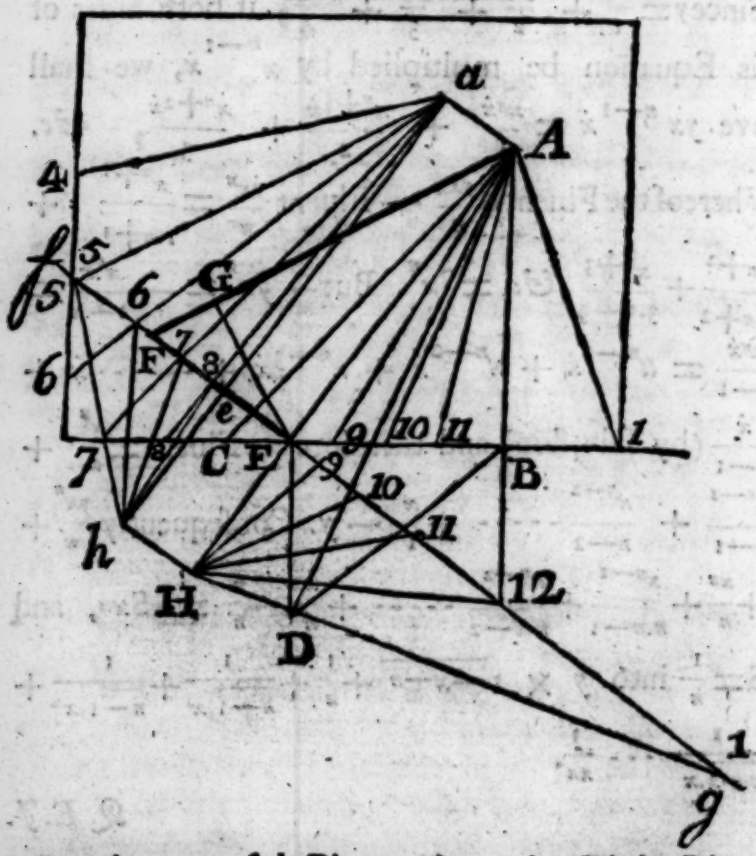
Q. E. I.

According to the ſame Method of proceeding, the Sums of a vaſt Number of other Series's may be determined: Which Room will not allow to be inſiſted on at preſent.

Problem VI.

To describe an erect South Dial, Geometrically, for any proposed North Latitude, to have a given Declination from the West; or an erect North Dial, for a South Latitude, to decline from the East.

Construction.



On the proposed Plane, draw the Right-Line BC for the Horizontal Line of the Dial, and perpendicular thereto draw BA; make the Angle BCA equal to the proposed Latitude, and the Angle CBD equal to the Complement of the given Declination; then, having taken BD equal to BC, draw DE perpendicular to BC: Join A, E and perpendicular thereto draw *f*Eg, in which set off

F.

Ee equal to the propoſed Thickneſs of your Style; thro' *e* draw *ea* perpendicular to *fg*, and let A*a* be parallel thereto: Make EF equal to ED, and joining A, F draw EG perpendicular thereto; in AE and *ae* produced take EH and *eb* each equal to EG, and from H to the Point where *fg* interſects AB, produced, draw H 12; then make the Angles 12 H 11, 11 H 10, 10 H 9 each equal to 15 Degrees, and the Angle *eb* 8 equal to the Exceſs of 15 Degrees above the Angle EH9: Alſo make the Angles 8*b*7, 7*b*6, 6*b*5, &c. each equal to 15 Degrees, and draw the Right Lines A11, A10, A9, *a*8, *a*7, *&c.* for the Hour Lines of 11, 10, 9, 8, *&c.*

To demonſtrate that the upper Edge of the Style, when the Dial is placed in its proper Poſition, will be parallel to the Earth's Axis (as it ought to be by Definition 2d) it may be obſerved that the Line BC being parallel to the Plane of the Horizon, the Line AB perpendicular thereto, as the Dial is an erect one, will be perpendicular to the ſaid Plane; therefore the Plane of the Triangle BDE, when erected perpendicular to the Plane of the Dial, on the Line BE, being perpendicular to the Line AB and parallel to the Plane of the Horizon, the Line BD will be perpendicular to the Plane of the Prime Vertical (becauſe the Angle DBE expreſſes the Complement of the Plane's Declination from the ſaid Plane, by Definition 12) and conſequently parallel to the Plane of the Meridian; therefore the Plane ADB, being alſo perpendicular to the Plane of the Prime Vertical, muſt coincide with the Plane of the Meridian.

Moreover, ſince DB is equal to BC (by Conſtruction) the Angles ABD and ABC both right Ones, and AB common, it is evident, that the Angle ADB is equal to the Angle ACB equal to

the given Latitude, and conſequently that the Line DA will be parallel to the Earth's Axis.

But the Triangle AEF, when the Plane of the Style, repreſented by the Triangle AFE, is erected perpendicular to That of the Dial, on the Subſtyle AE, will coincide with the Triangle ADE, becauſe DE is equal to FE, AE common and the Angles AFE, AED both Right.

If it be required to deſcribe an erect South Dial, for a Northern Latitude, to decline Weſterly; or an erect North Dial, for a Southern Latitude, to decline Eaſterly; the Method of Conſtruction will ſtill be the ſame, only, inſtead of beginning the Operation on the left Side of AB, you muſt begin on the right Side.

If the Plane of an erect South Dial, declining Eaſterly, in a Northern Latitude, be inverted, the Dial will then become an erect North Dial, declining Weſterly, for the ſame Latitude; and if the Plane of an erect North Dial declining Weſterly, in a Southern Latitude, be inverted, the Dial will then become an erect South Dial, declining Eaſterly, for the ſame Latitude.

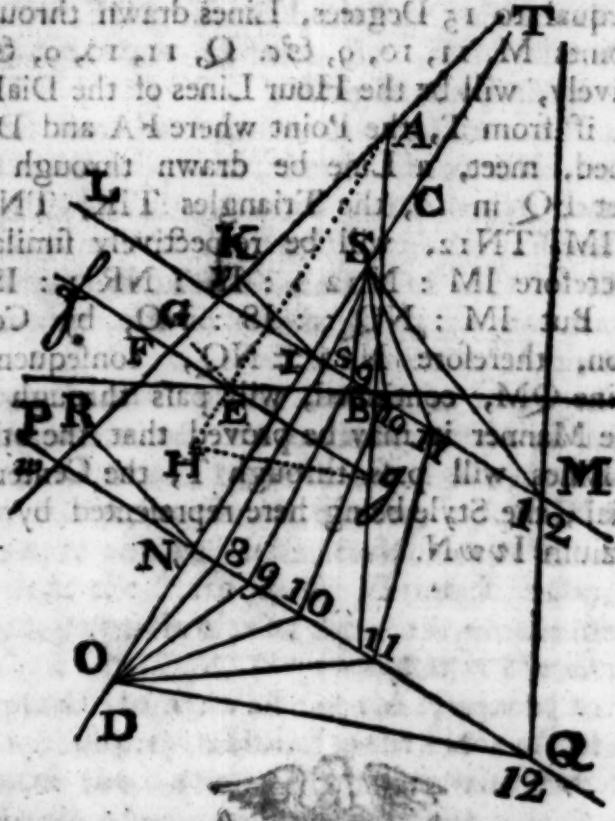
In like Manner, an erect South Dial, declining Weſterly, in a Northern Latitude, will become an erect North Dial declining Eaſterly; and an erect North Dial, declining Eaſterly, in a Southern Latitude, will become an erect South Dial, declining Weſterly.

If the Declination be very great, ſo that the Elevation of the Style, above the Subſtyle, becomes very ſmall, the preceding Method of Conſtruction will be impracticable; becauſe the Hour Lines, particularly Thoſe which fall near the Subſtyle, unleſs They be continued to a great Diſtance from the Center of the Dial, nearly coincide:

Which

Which Inconvenience may be removed by the following Manner of Conſtruction.

Having drawn the Style AF and the Subſtyle AE, as is taught above, let the Line EG be drawn



perpendicular to AF and, in AE continued, let EH be taken equal to EG; then, having drawn *f*E*g* perpendicular to AE produce AB to meet *f g* in *g* and join H, *g*. Moreover, at a convenient Diſtance, let the Line CD be drawn parallel to AE, for a new Subſtyle, and the Lines LM, PQ, at as great a Diſtance from each other as Room will permit, perpendicular thereto; alſo from the Points I, N where the Lines LM, PQ interſect CD,

CD, let IK, NR be drawn perpendicular to AF. Make IS equal to IK and NO equal to NR; alſo make the Angles ISM, NOQ each equal to the Angle EHg; then, if the Angles QO 11, MS 11, 11O 10, 11S 10, 10O 9, 10S 9, *&c.* be each made equal to 15 Degrees, Lines drawn through the Points M, 11, 10, 9, *&c.* Q, 11, 10, 9, *&c.* reſpectively, will be the Hour Lines of the Dial.

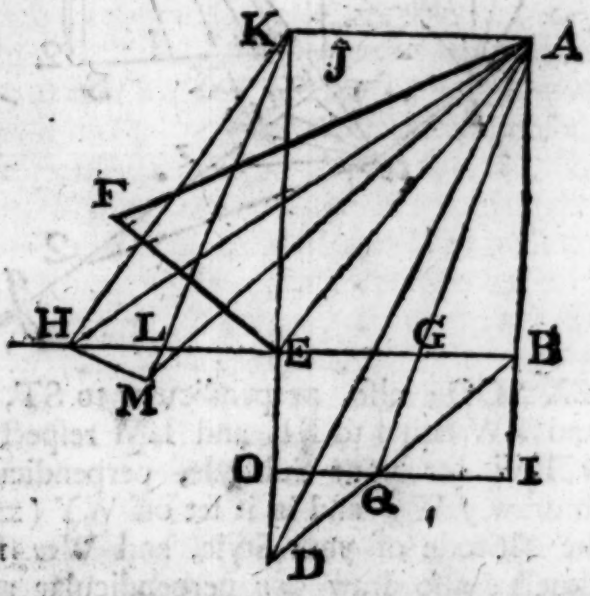
For, if from T, the Point where FA and DC, continued, meet, a Line be drawn through M to meet PQ in 12, the Triangles TIK, TNR, and TIM, TN12, will be reſpectively ſimilar; and therefore IM : N 12 : : IK : NR : : IS : NO: But IM : NQ : : IS : NO, by Conſtruction, therefore N 12 = NQ, conſequently the Line QM, continued, will paſs through T. In like Manner it may be proved that the other Hour Lines will paſs through T, the Center of the Dial; the Style being here repreſented by the Trapezium I*v*wN.

Problem VII.

To deſcribe an Eaſt Declining Dial, Geometrically, for any propoſed Northern Latitude, to have a given Reclination from the South; or a Weſt Declining Dial, for a Southern Latitude, to have a given Reclination from the North.

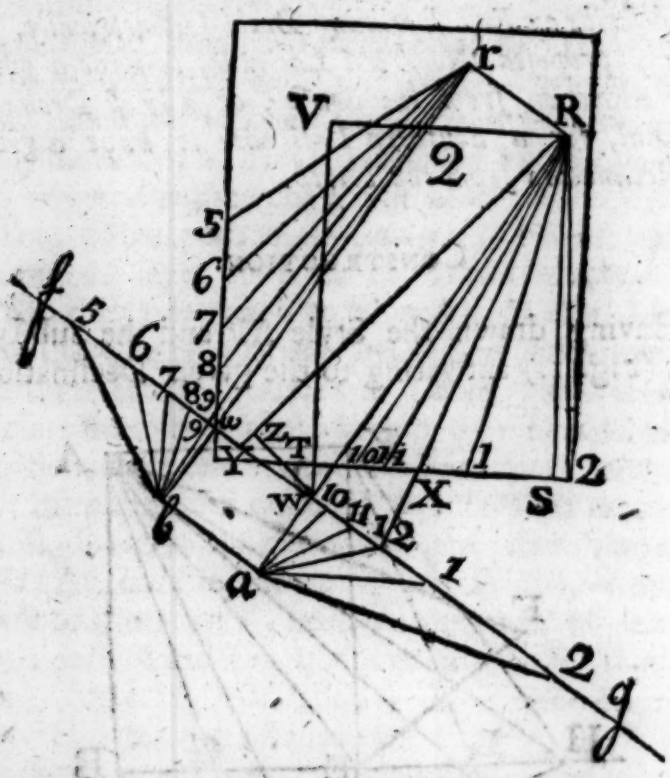
Construction.

Having drawn the Style AF and the Subſtyle AE (Fig. 1.) anſwering to the given Declination,



according to Problem VI. complete the Parallelogram ABEK, and in BE produced take EH = EF = ED; alſo join K, H and make the Angle EKL equal to the given Reclination. On KL, produced, let fall the Perpendicular HM and taking EO equal to EL, draw OI; parallel to EB, interſecting BD in Q.

On the propoſed Plane (Fig. 2.) draw SXT for the Horizontal Line, in which ſet off ST=BE



and TX=OQ; alſo, perpendicular to ST, draw SR and TW equal to KL and LM reſpectively: Draw RW for your Subſtyle, perpendicular to which draw *f* W *g* and in it ſet off WY (=HM) for the Altitude of your Style, and W*w* for its Thickneſs; alſo draw *w r* perpendicular and R*r* parallel to *f g*. In RW and *r w* produced take W*a* and *w b* each equal to WZ, perpendicular to RY, and through X draw RX12 meeting *f g* in 12; join *a*, 12 then make the Angles 12 *a* 11, 11 *a* 10 each equal to 15 Degrees, and the Angle *w b* 9 equal to the Exceſs of 15 Degrees above the Angle W*a* 10. Alſo make the Angles 9 *b* 8, 8 *b* 7, &c. each equal to 15 Degrees, then Right-

Lines

Lines drawn from R to 11, 10, 1, 2, and from *r* to 9, 8, 7, 6 will be the Hour Lines of 11, 10, 1, 2, 9, *&c.*

In Order to ſhew that the upper Edge of the Style, when the Dial is placed in its proper Poſition, will be parallel to the Earth's Axis (as it ought to be by Definition 2) it has been already proved, in the preceding Problem, that when the Plane ABEK (Fig. 1.) is perpendicular to the Plane of the Horizon, the Style FA is parallel to the Earth's Axis, in which Caſe (the Planes KEH, AEF and EBD being all perpendicular to the Plane ABEK) the Lines ED, EH and EF coincide and conſequently expreſs the Altitude of the Point F, of the Style, above the Plane ABEK. Now, as the Angle EKL expreſſes the given Reclination or the Inclination of the propoſed Plane to the Vertical Plane, if the Lines AH, AM be drawn, HM will expreſs the Altitude of the Point F above the Plane AKM and AH will coincide with AF and ſo be parallel to the Earth's Axis. But as SR (Fig. 2.) is by Conſtruction equal to KL, if the Parallelogram SV be completed, VW will be = KM the Triangle RVW equal and ſimilar to the Triangle AKM, and the Triangle RWY equal and ſimilar to the Triangle AMH; conſequently when the Plane of the Style, repreſented by the Triangle RWY, is erected perpendicular to the Plane of the Dial on the Line RW, the upper Edge (RY) thereof will paſs through the Pole of the World, and ſo be parallel to the Earth's Axis.

If the Dial be conſtructed for a Northern Latitude and EL be greater than EH, or, which is the ſame Thing, KL greater than KM, the North Pole will be elevated and the Point W,

in the Subſtyle RW, will fall on the contrary Side of R.

If it be required to deſcribe a Reclining South Dial, for a Northern Latitude, to decline Weſterly; or a Reclining North Dial, for a Southern Latitude, to decline Eaſterly; the Method of Conſtruction will ſtill be the ſame, only, inſtead of beginning the Operation on the left Side of AB and RS, you muſt begin on the right Side of Them.

In deſcribing a Reclining North Dial, for a North Latitude, to decline Eaſterly; or a Reclining South Dial, for a Southern Latitude to decline Weſterly; the Conſtruction differs from the former inſomuch that the Points A, B, E and K change their Poſition and fall in the Places of E, K, A and B reſpectively, and that the Line KL falls on the contrary Side of KE, with reſpect to the Line KH; alſo that the Points R, S, T and V fall in the Places of T, V, R and S reſpectively, and that the Hour Lines of 1, 2 become Thoſe of 11, 10, and Thoſe of 11, 10, 9, *&c.* become the Hour Lines of 1, 2, 3, *&c.*

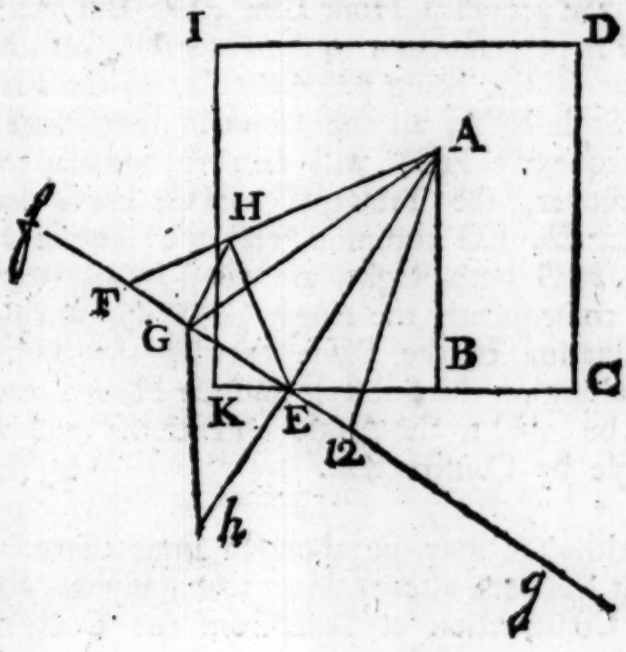
If the Plane of a Reclining South Dial, declining Eaſterly, in a Northern Latitude, be inverted; the Dial will then become an inclining North Dial, declining Weſterly, for the ſame Latitude; but the Morning Hour Lines in the Reclining Dial will become the Afternoon Hour Lines in the Inclining One. In like Manner a Reclining North Dial, declining Weſterly, in a Southern Latitude, will become an Inclining South Dial, declining Eaſterly, for the ſame Latitude. Alſo a Reclining South Dial, declining Weſterly, in a Northern Latitude, will become an Inclining North Dial, declining Eaſterly, for the ſame Latitude; and a Reclining North Dial, declining Eaſterly,

in

in a Southern Latitude, will become an Inclining South Dial, declining Weſterly, for the ſame Latitude. Moreover, a Reclining North Dial, declining Eaſterly, in a Northern Latitude, will become an Inclining South Dial, declining Weſterly, for the ſame Latitude; and a Reclining South Dial, declining Weſterly, in a Southern Latitude, will become an Inclining North Dial, declining Eaſterly, for the ſame Latitude.

Having now given the Conſtruction of the various Kinds of Dials, whether Erect, Declining, Reclining, or Inclining, I ſhall here put down the general Demonſtration of the Method, all along uſed, in drawing of the Hour Lines, which we promiſed in our firſt Problem, and thought would be better than taking up Room with the particular Demonſtration of each Problem, as they all depend upon the ſame Principle.

Let AE be the Subſtyle, AF the Style, A 12 the Meridian Line, and AG any other Hour Line



at

at Pleaſure; making (as is every where done) EH perpendicular to AF and Eh=EH; alſo let hG and GH be drawn.

Becauſe the Sun, in his apparent Diurnal Revolution, deſcribes an Angle of 15° each Hour about the Axis of the Earth, or the Style AF of the Dial, which is parallel to it, it is evident that if a plane, coinciding with (or lying upon) the Line AF, be ſuppoſed to revolve about the ſame as an Axis, at the Rate of 15° *per* Hour, it will when produced paſs thro' the Sun, and conſequently its Interſection with the plane of the Dial will mark out the true Poſition of the Hour Lines.

Now it is known that the Angle included between any two Planes is equal to (or meaſured by) the Angle made by two Lines in thoſe Planes perpendicular to their common Interſection: Alſo, by the general Method all along uſed, the Angle EhG is equal to the given Hour Angle betwixt the Time of the Planes falling upon the Subſtyle and the propoſed Hour Line AG: But the Line HG is perpendicular to the Axis AF, becauſe the Plane GHE, being perpendicular to the Plane of the Style EFA, all the Lines in that Plane (and conſequently HG) will be perpendicular to AF. Moreover, the Triangles EHG, E*b*G having EH=E*b*, EG common, and the Angles HEG and *b*EG both Right, are equal in all Reſpects, and conſequently the Angle GHE, meaſuring the Inclination of the Plane betwixt the Times of its falling on the Subſtyle and the Hour Line AG, will be equal to the Angle G*b*E = the ſaid Hour Angle by Conſtruction. *Q. E. D.*

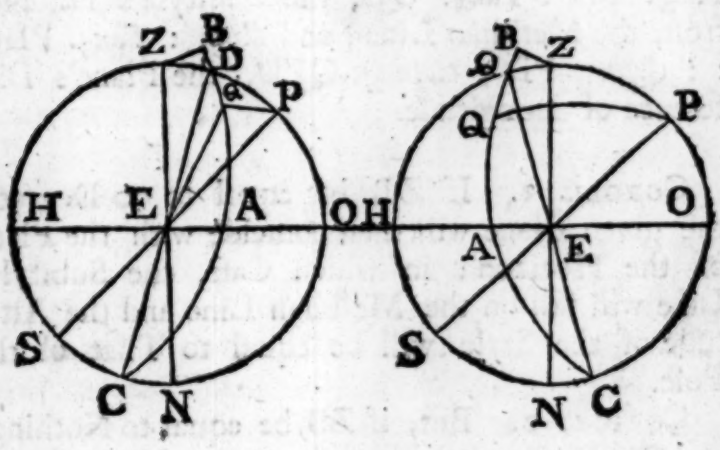
Altho' it may be thought unneceſſary, after what has been already done, to ſay any thing about the Conſtruction of Dials from the Doctrine of the

Sphere, eſpecially as this Part has been already conſidered by ſome good Authors; yet it may not be amiſs to give a few Pages on the Subject, as what we have to offer, tho' quite general, is comprized in little Compaſs and will nevertheleſs, I hope, be found ſufficiently plain and eaſy.

Problem I.

The Latitude of the Place, and the Poſition of the Plane being given; to determine the Meridian Line, the Subſtylar Line, the Height of the Style, and the Plane's Difference of Longitude. *

Let HZON be the Plane of the Meridian, BAC the given Plane, ZN the Prime Vertical,



HO the Horizon, and PS the Axis of the World; alſo let ZB and PQ be Arches of two Great Circles perpendicular to BAC; then will ZB be

* The Poſition of a Plane is ſaid to be given, when the Declination and Reclination of the Plane are given.

the

the Reclination and EA the Declination of the given Plane, alſo ED will be the Meridian Line, EQ the Subſtylar Line, QP or the Angle QEP the Height of the Style, DPQ the Plane's Difference of Longitude, and DA or the Angle DEA the Diſtance between the Meridian and Horizontal Lines.

Now, by the Principles of Spherical Trigonometry,

Rad. : *Sine*, BZD : : *Coſine*, BZ : *Coſine*, BDZ, *Rad.* : *Sine*, BZ : : *Tang.* BZD : *Tang.* BD (= BED), whence DA, or DEA, is given;

Rad : *Coſine*, BZ : : *Coſine*, BD : *Coſine*, DZ.

Then in the Right-angled Triangle PDQ, will be given PD (= ZD—PZ or ZD+PZ or ZP—ZD) and the Angle PDQ, whence

Rad. : *Sine*, BDQ : : *Sine*, PD : *Sine*, PQ, the Height of the Style; *Rad.* : *Coſine*, PDQ : : *Tang.* PD : *Tang.* QD, the Subſtyle's Diſtance from the Meridian Line; and *Rad.* : *Tang.* PDQ : : *Coſine*, PD : *Cotang.* QPD, the Plane's Difference of Longitude.

Coroll. 1. If ZB be equal to 90 Degrees, the given Plane will then coincide with the Plane of the Horizon; in which Caſe, the Subſtylar Line will fall on the Meridian Line and the Altitude of the Style will be equal to That of the Pole.

Coroll. 2. But, if ZB be equal to Nothing, and the Declination likewiſe equal to Nothing; the Plane will then coincide with the Prime Vertical; the Subſtylar Line will fall on the Meridian Line, Q will coincide with Z, the Altitude of the Style, will be equal to the Complement of Latitude, and the Planes Difference of Longitude equal to 90 Degrees.

Coroll.

Coroll. 3. Moreover, if ZB be equal to nothing and the Declination equal to 90 Degrees, the Plane will then be parallel to the Meridian; in which Caſe, the Meridian Line being indeterminable, the Subſtylar Line muſt be drawn parallel to the Axis of the World, and the Height of the Style may be taken at Pleaſure.

Coroll. 4. When ZB is equal to the Complement of the given Latitude, and the Declination equal to nothing, the Plane will become perpendicular to the Planes of the Equinoctial and of the Meridian of the given Place; the Meridian and Subſtylar Lines will then coincide, and the Altitude of the Style may be taken at Pleaſure.

Coroll. 5. If the Declination be equal to Nothing, and (B falling on the contrary Side of Z with Reſpect to P) ZB be equal to the Latitude of the Place, the Plane will then coincide with the Equinoctial and the Style will be perpendicular to the Plane.

Coroll. 6. If the Declination be equal to Nothing, the Plane will then become a direct inclining or reclining Plane, and the Arcs BZ and PQ will, each, coincide with the Meridian of the Place; whence it appears that the Height of the Style will always be equal to the Difference or Sum of the Reclination and of the Complement of Latitude, according as B falls on the ſame or the contrary Side of Z with Reſpect to P, and that the Meridian and Subſtylar Lines will coincide.

Coroll. 7. But if the Declination be equal to 90 Degrees, the Arc ZB will then fall in the Prime Vertical and ED will coincide with EO; therefore BDZ will be an Iſoſceles Triangle in

which the Sides are all given and the Angles B and Z both Right; whence, the firſt Proportions becoming Thoſe of Equality, we have, in the Right-angled Triangle PDQ,

Radius is to the Sine of the Reclination, as the Sine of the Latitude to the Sine of the Style's Altitude:

Radius is to the Coſine of the Reclination, as the Tangent of the Latitude to the Tangent of the Subſtyle's Diſtance from the Meridian: And

Radius is to the Tangent of the Reclination, as the Coſine of the Latitude to the Cotangent of the Inclination of the Meridians or of the Plane's Difference of Longitude.

Coroll. 8. When the Reclination ZB is equal to Nothing, or the Points B and D fall in Z, the Plane will be an erect declining one; and our Proportions will become,

Radius is to the Coſine of the Plane's Declination, as the Coſine of the Latitude to the Sine of the Style's Elevation:

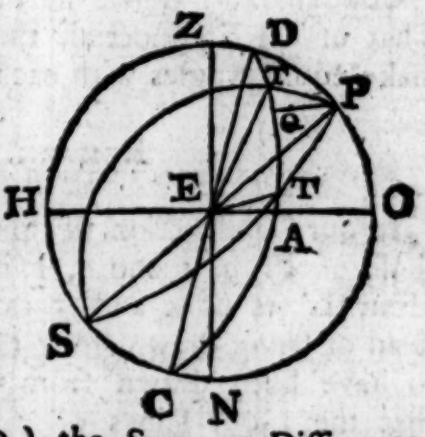
Radius is to the Sine of the Plane's Declination, as the Cotangent of the Latitude to the Tangent of the Subſtyle's Diſtance from the Meridian: And

Radius is to the Cotangent of the Plane's Declination, as the Sine of the Latitude to the Cotangent of the Inclination of the Meridians or of the Plane's Difference of Longitude.

Problem

Problem II.

The ſame being given as in the preceding Problem; to determine the Diſtance of the Hour Lines from the Subſtyle.



Let PTSEP be the Plane of an Hour Circle interſecting the given Plane DACE in the Line ET; then in the Triangle DPQ, we ſhall have Radius to the Sine of (PQ) the Style's Altitude, as the Tangent of (TPQ) the Sum, or Difference, of the given Hour Angle (DPT) and of (DPQ) the Planes Difference of Longitude, to the Tangent of (TQ) the Diſtance of the required Hour Line from the Subſtyle.

Coroll. 1. If the Plane coincides with the Horizon, the Proportion will become that of Radius to the Sine of Latitude, as the Tangent of the given Hour Angle to the Tangent of the Diſtance of the required Hour Line from the Subſtyle.

Coroll. 2. But if the Plane coincides with the Prime Vertical, Radius will be to the Coſine of the Latitude, as the Tangent of the given Hour Angle to the Tangent of the Diſtance of the Hour Line required from the Subſtyle.

Coroll.

COROLL. 3. Moreover, if the Plane coincides with the Meridian or any other Hour Circle, Radius will be to the Height of the Style, as the Tangent of the given Hour Angle to the Tangent of the Diſtance of the Hour Line ſought from the Subſtyle.

COROLL. 4. When the Plane coincides with That of the Equinoctial, the Hour Lines will make equal Angles with each other.

REMARK.

In our Conſtruction of ſome of the Dials, as thoſe of Prob. 1 and 4, it is evident that the Hour Lines of 4 and 5 in the Morning, inſtead of being drawn from the Center a, ought to have been drawn from the Center A; and that the Hour Lines of 7 and 8 in the Afternoon, inſtead of being drawn from the Center A, ought to have been drawn from the Center a.

For while the Sun is on the ſame Side of the Line Aa, with Reſpect to the Point C, the Hour Lines muſt be drawn from A or a, according as the Sun is on the ſame Side of AC, with Reſpect to the Points H and K, or on the contrary Side; the Shadow of that Side of the Style, correſponding to the Line AC, pointing out the Hour Lines in the former Caſe, and the Shadow of the Side, correſponding to the Line ac, pointing out the Hour Lines in the Latter.

The like Obſervation may be made, with Reſpect to other Dials, when the Sun appears in the Plane thereof, on the ſame Side of the Line Aa with Reſpect to the Point C.

FINIS.